ISSN: 2776-0960 Volume 2, Issue 8 August, 2021

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

$\widetilde{U}_0(x,\tau)$ ELLIPTIC PROBLEM SOLUTION ANALYTICITY WITH RESPECT TO τ. $\widetilde{U}_0^{(1)}(x,\tau)$ AND THEIR DERIVATIVES ASYMPTOTIC EVALUATIONS WITH τ

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Abstract

In the paper the $\widetilde{\boldsymbol{U}}_{\mathbf{0}}(\boldsymbol{x},\boldsymbol{\tau})$ elliptic problem $M(\partial_x)U - \nu\chi\theta - \chi^0\frac{\partial^2 U}{\partial t^2} = H,\ \Delta\theta - \frac{1}{\vartheta}\frac{\partial\theta}{\partial t} - \eta\frac{\partial}{\partial t}divu = H_7$ [1] is conducted to find a solution of the following two problems:

1) $M(\partial x)\widetilde{U}_0^{(1)}(x,\tau) - \tau^2\chi^0\widetilde{U}_0^{(1)}(x,\tau) = 0,\qquad \Delta\tilde{\theta}_0^{(1)}(x,\tau) - \frac{\tau}{\vartheta}\tilde{\theta}_0^{(1)}(x,t) = 0,$
$\lim\limits_{D\ni x\to y\in S}\widetilde{U}_0^{(1)}(x,\tau) = \tilde{f}_0(y,\tau),\ \lim\limits_{D\ni x\to y\in S}\tilde{\theta}_0^{(1)}(x,\tau) = \tilde{f}_{07}(y,\tau);$

2) 2) $M(\partial x)\widetilde{U}_0^{(2)}(x,\tau) - \tau^2\chi^0\widetilde{U}_0^{(2)}(x,\tau) - \upsilon\chi\tilde{\theta}_0^{(2)}(x,\tau) = \tilde{\vartheta}_0^{(1)}(x,\tau), \Delta\tilde{\theta}_0^{(2)}(x,\tau) - \frac{\tau}{\vartheta}\tilde{\theta}_0^{(2)}(x,\tau) - \eta\tau div\widetilde{U}_0^{(2)}(x,\tau) = \tilde{\vartheta}_{07}^{(1)}(x,\tau),\ \lim\limits_{D\ni x\to y\in S}\widetilde{U}_0^{(2)}(x,\tau) = 0.$

It is shown that $\widetilde{\boldsymbol{U}}_{\mathbf{0}}(\boldsymbol{x},\boldsymbol{\tau})$ elliptic problem's solution is analytic with respect to $\boldsymbol{\tau}$. $\widetilde{\boldsymbol{U}}_{\mathbf{0}}^{(\mathbf{1})}(\boldsymbol{x},\boldsymbol{\tau})$ and their derivatives asymptotic assessments with $\boldsymbol{\tau}$ are derived.

Key words: Elliptic problem, analyticity, asymptotic assessment, initial condition, boundary condition, integral equation.

Introduction

The main equations of Thermo-momentum Resiliency Dynamics can be written as

$$M(\partial x)U - \upsilon\chi\theta - \chi^0\frac{\partial^2 U}{\partial t^2} = \vartheta, \tag{1}$$

$$\Delta\theta - \frac{1}{\vartheta}\frac{\partial\theta}{\partial t} - \eta\frac{\partial}{\partial t}divu = \vartheta_7, \tag{2}$$

where $M(\partial x)$ is a differential operator of Resiliency Momentum Theory [4], [5].

$\chi = (\partial x_1, \partial x_2, \partial x_3, 0,0,0)$. $\chi^0 = \left\|\chi_{ij}^0\right\|_{6\times 6}$, $\chi_{ii}^0 = \rho$, $i = 1,2,3$, $\chi_{ii}^0 = Y$, $i = 4,5,6$. $\chi_{ii}^0 = 0$, $i \neq j$; $\vartheta = (-\rho\hbar, -\rho\lambda)$; $\vartheta_7 = -\frac{1}{\vartheta}Q$. $U = (u, w)$, where $u = (u_1, u_2, u_3)$ is a compatibility vector, $w = (w_1, w_2, w_3)$ – rotation vector, θ – temperature.

Let D be a finite or infinite three-dimensional space with the compact boundary S from the class $\Lambda 2(\alpha)$, $(\alpha > 0)$.

Denote by D_l and S_l cylinders $D_l = D \times l$, $S_l = S \times l$, respectively, where $l = [0, \infty)$.

Results

The following first problem is discussed: to find $\overline{D_l}$ cylinder's solutions $U = (u, \theta)$ of the equations (1) and (2) that belong to $C^1\left(\overline{D_l}\right) \cap C^2(D_l)$ and satisfy the initial conditions for $\forall x \in \overline{D}$:

$$\lim_{t\to 0} U(x,t) = \phi^{(0)}(x),\ \lim_{t\to 0}\theta(x,t) = \phi_7^{(0)}(x),\ \lim_{t\to 0}\frac{\partial U(x,t)}{\partial t} = \phi^{(1)}(x) \qquad (3)$$

and boundary conditions for $\forall (y,t) \in S_l$:

$$\lim_{D\ni x\to y\in S} U(x,t) = f(y,t),\ \lim_{D\ni x\to y\in S}\theta(x,t) = f_7(y,t). \qquad (4)$$

Here, $\phi^{(i)} = \left(\overset{1}{\phi}{}^{(i)}, \overset{2}{\phi}{}^{(i)}\right)$, $i = 0,1$, where $\overset{(k)}{\phi}{}^{(i)} = \left(\overset{k}{\phi}{}_1^{(i)}, \overset{k}{\phi}{}_2^{(i)}, \overset{3}{\phi}{}_i^{(k)}\right)$, $k = 1,2$ and $\phi_7^{(i)}$, $i = 0, 1$, are given functions in $\overline{D}$; $f = \left(f^{(1)}, f^{(2)}\right)$, $f^{(i)} = \left(f_1^{(i)}, f_2^{(i)}, f_3^{(i)}\right)$, $i = 1, 2$ and f_7 are given functions in S_l. Here, the initial and boundary conditions satisfy 1^0-5^0 conditions [3].

Let $U = (u, \theta)$ be a solution of the first problem, then $U_0 = U - H$, where $H = (h, h_7)$, $h^{(i)} = \left(h_1^{(i)}, h_2^{(i)}, h_3^{(i)}\right)$, $i = 1,2$, $h = e^{t^7}\sum_{k=0}^{6}\frac{t^k}{k!}\phi^{(k)}(x)$, $h^7 = e^{t^5}\sum_{k=0}^{4}\frac{t^k}{k!}\phi_7^{(k)}(x)$ (5)

are the solutions of (1) and (2) equations right-side part:

$$\vartheta_0 = \vartheta - M(\partial x)h + \upsilon\chi h_7 + \chi^0\frac{\partial^2 h}{\partial t^2},$$

$$\vartheta_{07} = \vartheta_7 - \Delta h_7 + \frac{1}{\vartheta}\frac{\partial}{\partial t}h_7 - \eta\frac{\partial}{\partial t}divh^{(1)}$$

with zero initial and boundary conditions:

$$\lim_{t\to 0} U(x,t) = 0,\ \lim_{t\to 0}\frac{\partial U_0(x,t)}{\partial t} = 0 \qquad (6)$$

$$\lim_{D\ni x\to y\in S} U_0(x,t) = f(y,t) - h(y,t) = f_0(y,t) \qquad (7)$$

$$\lim_{D\ni x\to y\in S}\theta_0(x,t) = f_7(y,t) - h_7(y,t) = f_{07}(y,t) \qquad (8)$$

Denote this problem by I^0.

I^0 problem, using the Laplace transform

$$\widetilde{U}_0(x,\tau)=\int_0^{\infty}e^{-\tau t}U_0(x,t)dt$$

is conducted to the elliptic problem:

$$M(\partial x)\widetilde{U}_0(x,\tau)-\chi^0\tau^2\widetilde{U}_0(x,\tau)-\upsilon\chi\tilde{\theta}_0(x,\tau)=\tilde{\vartheta}_0(x,\tau) \quad (9)$$

$$\Delta\tilde{\theta}_0-\frac{\tau}{\vartheta}\tilde{\theta}_0(x,\tau)-\eta\tau div\widetilde{U}_0(x,\tau)=\tilde{\vartheta}_{07}(x,\tau) \quad (10)$$

$$\lim_{D\ni x\to y\in S}\widetilde{U}_0(x,\tau)=\tilde{f}_0(y,\tau), \quad (11)$$

$$\lim_{D\in x\to y\in S}\tilde{\theta}_0(x,\tau)=\tilde{f}_{07}(y,\tau), \quad (12)$$

where

$$\tilde{\vartheta}_0(x,\tau)=\int_0^{\infty}e^{-\tau t}\vartheta_0(x,t)dt,\quad \tilde{\vartheta}_{07}(x,\tau)=\int_0^{\infty}e^{-\tau t}\vartheta_{07}(x,t)dt, \quad (13)$$

$$\tilde{f}_0(y,\tau)=\int_0^{\infty}e^{-\tau t}f_0(y,t)dt,\quad \tilde{f}_{07}(y,\tau)=\int_0^{\infty}e^{-\tau t}f_{07}(y,t)dt. \quad (14)$$

Let's denote this problem by (I_τ).

In order to show that for $\widetilde{U}_0(x,\tau)$ function there exists the Laplace inverse transform and this transform gives (I_τ) problem's classic solution, we have to derive evaluations of $\widetilde{U}_0(x,\tau)$ and its second order derivative with respect to τ. For this, (I_τ) problem's solution can be expressed as the sum of the following two problems:

1) $M(\partial x)\widetilde{U}_0^{(1)}(x,\tau)-\tau^2\chi^0\widetilde{U}_0^{(1)}(x,\tau)=0$,
$\Delta\tilde{\theta}_0^{(1)}(x,\tau)-\frac{\tau}{\vartheta}\tilde{\theta}_0^{(1)}(x,t)=0,\quad \lim_{D\ni x\to y\in S}\widetilde{U}_0^{(1)}(x,\tau)=\tilde{f}_0(y,\tau),\quad \lim_{D\ni x\to y\in S}\tilde{\theta}_0^{(1)}(x,\tau)=\tilde{f}_{07}(y,\tau)$;

2) $M(\partial x)\widetilde{U}_0^{(2)}(x,\tau)-\tau^2\chi^0\widetilde{U}_0^{(2)}(x,\tau)-\upsilon\chi\tilde{\theta}_0^{(2)}(x,\tau)=\tilde{\vartheta}_0^{(1)}(x,\tau), \Delta\tilde{\theta}_0^{(2)}(x,\tau)-\frac{\tau}{\vartheta}\tilde{\theta}_0^{(2)}(x,\tau)-\eta\tau div\widetilde{U}_0^{(2)}(x,\tau)=\tilde{\vartheta}_{07}^{(1)}(x,\tau),\quad \lim_{D\ni x\to y\in S}\widetilde{U}_0^{(2)}(x,\tau)=0.$ (15)

where

$$\tilde{\vartheta}_0^{(1)}(x,\tau)=\tilde{\vartheta}_0(x,\tau)+\left(\tau^2-\tilde{Y}^2\right)\chi^2\widetilde{U}_0^{(1)}(x,\tau)+\upsilon\chi\tilde{\theta}_0^{(1)}(x,\tau), \quad (16)$$

$$\tilde{\vartheta}_{07}(x,\tau)=\tilde{\vartheta}_{07}(x,\tau)+\frac{\tau-\tilde{Y}}{\vartheta}\tilde{\theta}_0^{(1)}(x,\tau)+\eta\tau div\widetilde{U}_0^{(1)}(x,\tau); \quad (17)$$

and $\tilde{Y}$is a fixed real number more than $\tilde{Y}^2$ [6].

Denote

$$\widetilde{U}_0^{(1)}=\left(\widetilde{U}_0^{(1)},\tilde{\theta}_0^{(1)}\right),\ \widetilde{U}_0^{(2)}=\left(\widetilde{U}_0^{(2)},\tilde{\theta}_0^{(2)}\right),$$

Let, $\tau\in\Pi_{Y_0}$ $\left(\Pi_{Y_0}=\{\tau=Y+i\xi\in Z|Y\geq Y_0'>Y_0\}\right)$.

Using the integration by parts formula for (13) and (14), from the following qualities:

$$\left(\frac{\partial^m \vartheta_0}{\partial t^m}\right)_{t=0} = 0, \qquad \left(\frac{\partial^k \vartheta_{07}}{\partial t^k}\right)_{t=0} = 0,\ x \in \overline{D} \tag{18}$$

$$\left(\frac{\partial^m f_0(y,t)}{\partial t^m}\right)_{t=0} = 0, \quad \left(\frac{\partial^k f_{07}(y,t)}{\partial t^k}\right)_{t=0} = 0,\ y \in S \tag{19}$$

$$m = 0,1,\dots,6,\ k = 0,1,\dots,4$$

we can derive

$$\tilde{\vartheta}_0(x,\tau) = \frac{1}{\tau^6}\left[\frac{\partial^5 \vartheta_0(x,t)}{\partial t^5}\right]_{t=0} + \frac{1}{t^6}\int_0^\infty e^{-\tau t}\frac{\partial^6 \vartheta_0(x,t)}{\partial t^6}dt, \tag{20}$$

$$\tilde{\vartheta}_{07}(x,\tau) = \frac{1}{\tau^5}\left[\frac{\partial^4 \vartheta_{07}(x,t)}{\partial t^4}\right]_{t=0} + \frac{1}{\tau^5}\int_0^\infty e^{-\tau t}\frac{\partial^5 \vartheta_{07}(x,t)}{\partial t^5}dt, \tag{21}$$

$$\tilde{f}_0(y,\tau) = \frac{1}{\tau^7}\int_0^\infty e^{-\tau t}\frac{\partial^7 f_0(y,t)}{\partial t^7}dt, \tag{22}$$

$$\tilde{f}_{07}(y,\tau) = \frac{1}{\tau^5}\int_0^\infty e^{-\tau t}\frac{\partial^5 f_{07}(y,t)}{\partial t^5}dt \tag{23}$$

$$f_0(y,\tau) = \frac{1}{\tau^8}\left[\frac{\partial^7 f_0(y,t)}{\partial t^7}\right]_{t=0} + \frac{1}{\tau^8}\int_0^\infty e^{-\tau t}\frac{\partial^8 f_0(y,t)}{\partial t^8}dt \tag{24}$$

$$f_{07}(y,\tau) = \frac{1}{\tau^6}\left[\frac{\partial^5 f_{07}(y,t)}{\partial t^5}\right]_{t=0} + \frac{1}{\tau^6}\int_0^\infty e^{-\tau t}\frac{\partial^6 f_{07}(y,t)}{\partial t^6}dt \tag{25}$$

From these qualities, considering 10-50 [1], we can derive that

$$\tilde{\vartheta}_0(x,\tau) \in C^{1,\delta}(\overline{D}), \tilde{\vartheta}_0(x,\tau) \in C^{1,\delta}(\overline{D}), \tag{26}$$

$$\tilde{f}_0(y,\tau) \in C^{1,\lambda}(S),\ \tilde{f}_{07}(y,\tau) \in C^{1,\lambda}(S). \tag{27}$$

In Π_{Y_0} semi plane the following conditions [1] are true:

$$\left\|\tilde{\vartheta}_0(x,\tau)\right\|_{(\overline{D},0,\delta)} \le \frac{C}{|\tau^6|}, \qquad \left\|\tilde{\vartheta}_{07}(x,\tau)\right\|_{(\overline{D},0,\delta)} \le \frac{C}{|\tau|^5}, \tag{28}$$

$$\left\|\tilde{f}_0(y,\tau)\right\|_{(S,0,\lambda)} \le \frac{C}{|\tau|^8}, \quad \left\|\tilde{f}_{07}(y,\tau)\right\|_{(S,0,\lambda)} \le \frac{1}{|\tau|^6}, \tag{29}$$

$$\left\|\tilde{f}_0(y,\tau)\right\|_{(S,1,\lambda)} \le \frac{C}{|\tau|^7}, \qquad \left\|\tilde{f}_{07}(y,\tau)\right\|_{(S,1,\lambda)} \le \frac{C}{|\tau|^6}, \tag{30}$$

For the highest meaning of $|x|$, using 50 condition [3], we have

$$\left|\tilde{\vartheta}_0(x,\tau)\right| \le \frac{C}{|x|^2}\frac{1}{|\tau|^6}, \left|\tilde{\vartheta}_{07}(x,\tau)\right| \le \frac{C}{|x|^2}\frac{1}{|\tau|^5}. \tag{31}$$

The first problem has a unique solution [7] and can be expressed as

$$\widetilde{U}_0^{(1)}(x,\tau) = \int_S \left[T(\partial y, n(y))\Psi'(x-y, i\tilde{Y})\right]' \Psi(y,\tau)dyS. \tag{32}$$

where $T(\partial y, n(y))$ is an operator of a tension moment, $n(y)$ is a normal of S surface in the point y. $\Psi(x-y, i\tilde{Y})$ equation is a fundamental matrix of the first problem [7].

Ψ vector-function for the inner problem gives the solution of the following integral equation:

$$-\psi(z,\tau)+\int_S\left[T\big(\partial y,n(y)\big)\Psi\big(z-y,i\tilde{Y}\big)\right]'\psi(y,\tau)dyS=\tilde{f}_0(z,\tau), \qquad (33)$$

and for the outer problem the following equation:

$$\psi(z,\tau)+\int_S\left[T\big(\partial y,n(y)\big)\Psi'\big(z-y,i\tilde{Y}\big)\right]'\psi(y,\tau)dyS=\tilde{f}_0(z,\tau). \qquad (34)$$

The solution of this equation in Π_{Y_0} semi plane is analytic with respect to τ, as the right side of this integral equation is an analytic function of τ . Therefore, $U_0^{(1)}$ is analytically dependent on τ.

The Banach theorem shows that the equations (33) and (34) corresponding operator in $C^{0,\lambda}(S)$ set has the inverse operator. Therefore,

$$\|\psi(\cdot,\tau)\|_{(S,0,\lambda)}\le C\cdot\left\|\tilde{f}_0(\cdot,\tau)\right\|_{(S,0,\lambda)}.$$

From which, based on (29), we have

$$\|\psi(\cdot,\tau)\|_{(S,0,\lambda)}\le\frac{C}{|\tau|^8}. \qquad (35)$$

Analogously, based on (30), we have

$$\|\psi(\cdot,\tau)\|_{(S,1,\lambda)}\le\frac{C}{|\tau|^7}. \qquad (36)$$

Using (35) and (36), from (32) [7] for $U_0^{(1)}$ we get the assessment:

$$\left\|\widetilde{U}_0^{(1)}(\cdot,\tau)\right\|_{\left(\overline{D}^{\pm},0,\lambda\right)}\le C\cdot\|\psi(\cdot,\tau)\|_{(S,0,\lambda)}\le\frac{C}{|\tau|^8} \qquad (37)$$

$$\left\|\widetilde{U}_0^{(1)}(\cdot,\tau)\right\|_{\left(\overline{D}^{\pm},1,\lambda\right)}\le\frac{C}{|\tau|^7}. \qquad (38)$$

For the inner area, based on the inequality [6]

$$|\partial^p\Phi(x,i\tau)|\le\frac{C}{|x|^{p+1}}|\tau|^p,\ \ |p|=0,1,\ldots$$

and from (35), (36), (32), we get:

$$\left|\widetilde{U}_0^{(1)}(x,\tau)\right|\le\frac{C}{|x|^2}\frac{1}{|\tau|^8}, \qquad (39)$$

$$\left|\frac{\partial}{\partial x_k}\widetilde{U}_0^{(1)}(x,\tau)\right|\le\frac{C}{|x|^2}\frac{1}{|\tau|^8}, \qquad (40)$$

where $k=1,2,3,\ \tau\in\Pi_{Y_0}$.

For any area $\overline{D}^*\subset D$ we have:

$$\sup_{x\in D^*}\left|\frac{\partial^2}{\partial x_k\partial x_j}\widetilde{U}_0^{(1)}(x,\tau)\right|\le\frac{C(D^*)}{|\tau|^8},\ \tau\in\Pi_{Y_0}, \qquad (41)$$

This means that there exists one unique solution for the first problem [1] that can be expressed as

$$\tilde{\theta}_0^{(1)}(x,\tau)=\int_S\frac{\partial}{\partial n(y)}\frac{e^{-k|x-y|}}{|x-y|}\phi_7(y,\tau)dyS,\ x\in D \qquad (42)$$

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where $k = \frac{\tilde{Y}}{\vartheta} > 0$, and $\phi_7(x,\tau)$ function represents a solution of the singular integral equation:

$$\phi_7(z,\tau) + \frac{1}{2\pi}\int_S \frac{\partial}{\partial n(y)} \frac{e^{-k|z-y|}}{|z-y|} \phi_7(y,\tau) dyS = \frac{1}{2\pi}\tilde{f}_{07}(z,\tau) \tag{43}$$

for the inner problem, and

$$-\phi_7(z,\tau) + \frac{1}{2\pi}\int_S \frac{\partial}{\partial n(y)} \frac{e^{-k|z-y|}}{|z-y|} \phi_7(y,\tau) dyS = \frac{1}{2\pi}\tilde{f}_{07}(z,\tau) \tag{44}$$

for the outer problem.

As $\tilde{f}_{07}(z,\tau)$ is analytic with respect to τ, the solutions of (43) and (44) will be also analytic functions with respect to τ. So, from (42), $\tilde{\theta}_0^{(1)}(x,\tau)$ will be analytically dependent on τ.

For $\tau \in \Pi_{Y_0}$, using (29) and (31), from (43) and (44), based on the Banach theorem, we get:

$$\|\phi_7(\cdot,\tau)\|_{(S,0,\lambda)} \leq C\left\|\tilde{f}_{07}(\cdot,\tau)\right\|_{(S,0,\lambda)} \leq \frac{C}{|\tau|^6}, \tag{45}$$

$$\|\phi_7(\cdot,\tau)\|_{(S,1,\lambda)} \leq \frac{C}{|\tau|^5}. \tag{46}$$

Hence, from (42) we get [7]:

$$\left\|\tilde{\theta}_0^{(1)}(\cdot,\tau)\right\|_{\left(\overline{D}^+,0,\lambda\right)} \leq C\|\phi_7(\cdot,\tau)\|_{(S,1,\lambda)} \leq \frac{C}{|\tau|^6}, \tag{47}$$

$$\left\|\tilde{\theta}_0^{(1)}(\cdot,\tau)\right\|_{\left(\overline{D}^+,1,\lambda\right)} \leq C\|\phi_7(\cdot,\tau)\|_{(S,1,\lambda)} \leq \frac{C}{|\tau|^5}, \tag{48}$$

$$\left|\frac{\partial}{\partial x_k}\tilde{\theta}_0^{(1)}(x,\tau)\right| \leq \frac{C}{|\tau|^5},\ x \in \overline{D}^+,\ k = 1,2,3 \tag{49}$$

For the highest $|x|$ from (46) we can derive:

$$\left|\tilde{\theta}_0^{(1)}(x,\tau)\right| \leq \frac{C}{|x|^2}\cdot\frac{1}{|\tau|^6},\ \left|\frac{\partial\tilde{\theta}_0^{(1)}(x,\tau)}{\partial x_k}\right| \leq \frac{C}{|x|^2}\frac{1}{|\tau|^5},\ k = 1,2,3. \tag{50}$$

For $\overline{D}^* \subset D$

$$\sup_{x\in D^*}\left|\frac{\partial^2}{\partial x_k \partial x_j}\tilde{\theta}_0^{(1)}(x,\tau)\right| \leq \frac{C(D^*)}{|\tau|^6},\ k,j = 1,2,3. \tag{51}$$

Conclusion

It is proved that $\tilde{U}_0^{(1)}(x,\tau)$=$\left(\tilde{U}_0^{(1)},\tilde{\theta}_0^{(1)}\right)$ is analytic with respect to τ. The asymptotic assessments of $\tilde{\boldsymbol{U}}_0^{(1)}(\boldsymbol{x},\boldsymbol{\tau})$ and their derivatives are derived in order to get integral equations.

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