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APPLICATION OF LINEAR ALGEBRA TO ECONOMIC QUESTIONS

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Abstract

In the analysis of intersectoral balancing model economic problems, Kramer's rule, Gaussian method, and matrix methods are commonly utilized. The inverse matrix method was used to tackle the three-sector economic system's balance problems in particular. The dynamic model of intersectoral balance deals with challenges that arise over a period of time, such as one year of manufacturing.

Introduction

The gross output of a network is represented by a system of equations X = AX + Y in each equation of static ratios, which is equal to the total of products consumed by the industry and other industries, as well as net production, i.e. the product of marginal usage. The matrices X, Y, and A are as follows.

$$X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

Main Part

If the matrix of the annual finite consumption product X ^ ((t)) is Z ^ ((t)) = B ^ ((t)) · (X ^ ((t)) -X ^ ((t-1))) If the investment and the marginal consumption product are expressed as the sum of Y ^ ((t)), then the dynamic model is as follows:

$$X^{(t)} = AX^{(t)} + B^{(t)}\left(X^{(t)} - X^{(t-1)}\right) + Y^{(t)} \qquad (1)$$

Here, the index t above each of the parentheses characterizes the number of the year, and the matrix B ^ ((t)) characterizes the network-to-network investment and is entered in the same way as the direct material cost matrix. The elements of A matrix $a_{ij} = \frac{x_{ij}}{x_j}$ determined by the formula. Similarly, the elements of the matrix B ^ ((t)) are defined in this form (2), i.e.

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$$b_{ij}^{(t)} = \frac{z_{ij}^{(t)}}{x_j^{(t)} - x_j^{(t-1)}} \qquad (2)$$

Here $z_{ij}^{(t)}$-$B^{(t)}$ the matrix element represents the product i sent to the network j for investment purposes.

In the dynamic model of intersectoral balance, the model indicators for the end of the period under t zeros ranging from 1 to T are considered. The starting condition is X ^ ((0)) production in zero years. Y ^ ((t)) can be expressed as Equation (1) in a n$\left(E - A^{(t)} - B^{(t)}\right) \cdot X^{(t)} = Y^{(t)} - B^{(t)} \cdot X^{(t-1)}$ given finite consumption.

The solution to this equation is as follows:

$$X^{(t)} = \left(E - A^{(t)} - B^{(t)}\right)^{-1} \cdot \left(Y^{(t)} - B^{(t)} \cdot X^{(t-1)}\right) \quad (3.2.3)$$

As a result of this formula, increasing production X ((t)) necessitates increasing marginal consumption Y ((t)) in comparison to the previous year.

The product of two matrices B ^ ((t)) · (X ^ ((t)) -X ^ ((t-1)))

$$Z^{(t)} = B^{(t)} \cdot \left(X^{(t)} - X^{(t-1)}\right)$$

we write the investment in a matrix-column view. In this case, for a given production and finite consumption, the formula for this matrix-column can be found from Equation (1).

Then the formula is: Z ^ ((t)) = (E-A) X ^ ((t)) -Y ^ ((t)).

In the real economy cross-sectoral balance model, of course, there are limiting factors. One such factor is the unequal distribution of labor resources.

$$l \cdot X^{(t)} \leq L^{(t)}$$

Here L ^ ((t)) - labor resources, l - labor capacity or labor costs per unit of output. This constraint can be given by $\bar{X}^{(t)} \leq \bar{X}^{(t-1)} + Z^{(t)}$. In the dynamic model considered here, it is assumed that the power of the networks will be fully utilized during the period under consideration. Therefore, in times of crisis or in a transition economy, Equation (1) should be replaced by a system of inequalities that represents, for example, the current production costs of gross output, the costs of expanding production capacity, and non-production costs. The gross output of the industries should not exceed the production capacity and available labor resources.

Example. From the data in the table, calculate the cross-sectoral balance parameters for the first and second years, for example, which are taken as the initial conditions of the problem. In this case, as non-production consumption:

Manufacturing industries	Istemol tarmoqlari			
	Agricultural sectors	Industrial networks	Industrial networks	General version
Agricultural sectors	50	40	110	200
Industrial networks	70	30	150	250
Industrial networks	80	180	40	300
General version	200	250	300	

1- variant: $Y^{(1)} = \begin{pmatrix}110\\150\end{pmatrix}$; ***2-variant:*** $Y^{(1)} = \begin{pmatrix}120\\160\end{pmatrix}$, $Y^{(2)} = \begin{pmatrix}130\\170\end{pmatrix}$ to be accepted. Assume that the matrix B ^ ((t)), which characterizes the investment from network to network, is time-independent and equal to

$$B^{(t)} = B = \begin{pmatrix}0{,}06 & 0{,}02\\0{,}04 & 0{,}1\end{pmatrix}$$

Solution. From the data in the table, the gross product from year zero is determined by the matrix-column $X^{(0)} = \begin{pmatrix}200\\250\end{pmatrix}$. In the correct material state, the elements of the matrix A are found by the formula $a_{ij} = \frac{x_{ij}}{x_j}$. For example, and so on $a_{11} = \frac{x_{11}}{x_1} = \frac{50}{200} = 0{,}25$. Thus, the correct material cost matrix is:

$$A = \begin{pmatrix}0{,}25 & 0{,}16\\0{,}35 & 0{,}12\end{pmatrix}$$

Option 1: The product produced in the first year

$$X^{(t)} = AX^{(t)} + B^{(t)}\left(X^{(t)} - X^{(t-1)}\right) + Y^{(t)} \quad (1)$$

by the formula Putting the data given in (3.2.1)

$$X^{(1)} = \begin{pmatrix}1-0{,}25-0{,}06 & -0{,}16-0{,}02\\-0{,}35-0{,}04 & 1-0{,}12-0{,}1\end{pmatrix}^{-1} \cdot \left(\begin{pmatrix}110\\150\end{pmatrix} - \begin{pmatrix}0{,}06 & 0{,}02\\0{,}04 & 0{,}1\end{pmatrix} \cdot \begin{pmatrix}200\\250\end{pmatrix}\right) =$$

$$= \begin{pmatrix}0{,}69 & -0{,}18\\-0{,}39 & 0{,}78\end{pmatrix}^{-1} \cdot \left(\begin{pmatrix}110\\150\end{pmatrix} - \begin{pmatrix}17\\33\end{pmatrix}\right) = \begin{pmatrix}0{,}69 & -0{,}18\\-0{,}39 & 0{,}78\end{pmatrix}^{-1} \cdot \begin{pmatrix}93\\117\end{pmatrix} = \begin{pmatrix}200\\250\end{pmatrix}$$

we get the result. Production in the first year is equal to production in year zero. This result was expected because the table shows the balance sheet without taking into account the investment.

$$.Z^{(1)} = (E - A) \cdot X^{(1)} - Y^{(1)}$$

This result and the data obtained can be verified by putting it in a formula

$$Z^{(1)} = \begin{pmatrix} 1 - 0,25 & -0,16 \\ -,035 & 1 - 0,12 \end{pmatrix} \cdot \begin{pmatrix} 200 \\ 250 \end{pmatrix} - \begin{pmatrix} 110 \\ 150 \end{pmatrix} = \begin{pmatrix} 110 \\ 150 \end{pmatrix} - \begin{pmatrix} 110 \\ 150 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

So, bottom line is that we're really looking forward to the second year.

2-variant: Option 2: In this option, unlike the previous one, non-productive consumption is determined by the column matrix $Y^{(1)} = \begin{pmatrix} 120 \\ 160 \end{pmatrix}$. In this case, production from the first year looks like this:

$$X^{(1)} = \begin{pmatrix} 1 - 0,25 - 0,06 & -0,16 - 0,02 \\ -0,35 - 0,04 & 1 - 0,12 - 0,1 \end{pmatrix}^{-1} \cdot \left(\begin{pmatrix} 120 \\ 160 \end{pmatrix} - \begin{pmatrix} 0,06 & 0,02 \\ 0,04 & 0,1 \end{pmatrix} \cdot \begin{pmatrix} 200 \\ 250 \end{pmatrix} \right) =$$

$$= \begin{pmatrix} 0,69 & -0,18 \\ -0,39 & 0,78 \end{pmatrix}^{-1} \cdot \left(\begin{pmatrix} 120 \\ 160 \end{pmatrix} - \begin{pmatrix} 17 \\ 33 \end{pmatrix} \right) = \begin{pmatrix} 0,69 & -0,18 \\ -0,39 & 0,78 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 103 \\ 127 \end{pmatrix} = \begin{pmatrix} 220,51 \\ 273,08 \end{pmatrix}.$$

Investment matrix column will be equaled

$$Z^{(1)} = \begin{pmatrix} 1 - 0,25 & -0,16 \\ -,035 & 1 - 0,12 \end{pmatrix} \cdot \begin{pmatrix} 220,51 \\ 273,08 \end{pmatrix} - \begin{pmatrix} 120 \\ 160 \end{pmatrix} = \begin{pmatrix} 121,69 \\ 163,13 \end{pmatrix} - \begin{pmatrix} 120 \\ 160 \end{pmatrix} = \begin{pmatrix} 1,69 \\ 3,13 \end{pmatrix}$$

The element of the matrix B, which represents the product sent by $z_{ij}^{(t)}$ the i-sector to the j-sector for investment purposes, is found from the relation (2).This element can be find in this formula $z_{ij}^{(t)} = b_{ij}(x_j^{(t)} - x_j^{(t-1)})$.

First we find the accuracy between the columns of the production matrix in the first and zero years:

$$X^{(1)} - X^{(0)} = \begin{pmatrix} 220,51 \\ 273,08 \end{pmatrix} - \begin{pmatrix} 200 \\ 250 \end{pmatrix} = \begin{pmatrix} 20,51 \\ 23,08 \end{pmatrix}$$

using this information we create.

$$z_{11}^{(1)} = b_{11} \left(x_1^{(1)} - x_1^{(0)} \right) = 0,06 \cdot 20,51 = 1,23;$$

$$z_{12}^{(1)} = b_{12} \left(x_2^{(1)} - x_2^{(0)} \right) = 0,02 \cdot 23,08 = 0,46;$$

$$z_{21}^{(1)} = b_{21} \left(x_1^{(1)} - x_1^{(0)} \right) = 0,04 \cdot 20,51 = 0,82;$$

$$z_{22}^{(1)} = b_{22} \left(x_2^{(1)} - x_2^{(0)} \right) = 0,1 \cdot 20,08 = 2,31;$$

We will check the results by placing them on the right side of the given formula.

$$X^{(1)} = AX^{(1)} + B^{(1)}\left(X^{(1)} - X^{(0)}\right) + Y^{(1)}$$

$$\begin{pmatrix} 0{,}25 & 0{,}16 \\ 0{,}35 & 0{,}12 \end{pmatrix} \cdot \begin{pmatrix} 220{,}51 \\ 273{,}08 \end{pmatrix} + \begin{pmatrix} 0{,}06 & 0{,}02 \\ 0{,}04 & 0{,}1 \end{pmatrix} \cdot \begin{pmatrix} 20{,}51 \\ 23{,}08 \end{pmatrix} + \begin{pmatrix} 120 \\ 160 \end{pmatrix} =$$

$$= \begin{pmatrix} 98{,}82 \\ 109{,}95 \end{pmatrix} + \begin{pmatrix} 1{,}69 \\ 3{,}13 \end{pmatrix} + \begin{pmatrix} 120 \\ 160 \end{pmatrix} = \begin{pmatrix} 220{,}51 \\ 273{,}08 \end{pmatrix}$$

After the calculations, we get exactly the same amount as in the first year of production. For the second year, the non-production consumption matrix is determined by a column. In that case the production of the second year

$$Y^{(2)} = \begin{pmatrix} 130 \\ 170 \end{pmatrix}$$

$$X^{(2)} = \begin{pmatrix} 1 - 0{,}25 - 0{,}06 & -0{,}16 - 0{,}02 \\ -0{,}35 - 0{,}04 & 1 - 0{,}12 - 0{,}1 \end{pmatrix}^{-1}$$

$$\cdot \left(\begin{pmatrix} 130 \\ 170 \end{pmatrix} - \begin{pmatrix} 0{,}06 & 0{,}02 \\ 0{,}04 & 0{,}1 \end{pmatrix} \cdot \begin{pmatrix} 220{,}51 \\ 273{,}08 \end{pmatrix} \right) =$$

$$= \begin{pmatrix} 0{,}69 & -0{,}18 \\ -0{,}39 & 0{,}78 \end{pmatrix}^{-1} \cdot \left(\begin{pmatrix} 130 \\ 170 \end{pmatrix} - \begin{pmatrix} 18{,}69 \\ 36{,}13 \end{pmatrix} \right) = \begin{pmatrix} 0{,}69 & -0{,}18 \\ -0{,}39 & 0{,}78 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 111{,}31 \\ 133{,}87 \end{pmatrix}$$

$$= \begin{pmatrix} 237{,}00 \\ 290{,}13 \end{pmatrix}$$

Investment matrix column consists of

$$Z^{(2)} = \begin{pmatrix} 1 - 0{,}25 & -0{,}16 \\ -{,}035 & 1 - 0{,}12 \end{pmatrix} \cdot \begin{pmatrix} 237{,}00 \\ 290{,}13 \end{pmatrix} - \begin{pmatrix} 130 \\ 170 \end{pmatrix} = \begin{pmatrix} 131{,}34 \\ 172{,}36 \end{pmatrix} - \begin{pmatrix} 130 \\ 170 \end{pmatrix} = \begin{pmatrix} 1{,}34 \\ 2{,}36 \end{pmatrix}$$

The matrix element B (2), which represents the product $z_{ij}^{(t)}$ sent from the i-th network to the j-th network for investment purposes, is found from the relation (2). This element is found from the formula $z_{ij}^{(t)} = b_{ij}(x_j^{(t)} - x_j^{(t-1)}$.

First we find the difference between the production matrix-columns of the second and first year:

$$X^{(2)} - X^{(1)} = \begin{pmatrix} 237{,}00 \\ 290{,}13 \end{pmatrix} - \begin{pmatrix} 220{,}51 \\ 273{,}08 \end{pmatrix} = \begin{pmatrix} 16{,}49 \\ 17{,}05 \end{pmatrix}$$

using this information we get this results

$$z_{11}^{(2)} = b_{11}\left(x_1^{(2)} - x_1^{(1)}\right) = 0{,}06 \cdot 16{,}49 = 0{,}99;$$

$$z_{12}^{(2)} = b_{12}\left(x_2^{(2)} - x_2^{(1)}\right) = 0{,}02 \cdot 17{,}05 = 0{,}341;$$

$$z_{21}^{(2)} = b_{21}\left(x_1^{(2)} - x_1^{(1)}\right) = 0{,}04 \cdot 16{,}49 = 0{,}66;$$

$$z_{22}^{(2)} = b_{22}\left(x_2^{(2)} - x_2^{(1)}\right) = 0{,}1 \cdot 17{,}05 = 1{,}705$$

And write $X^{(2)} = AX^{(2)} + B^{(2)}(X^{(2)} - X^{(1)}) + Y^{(2)}$ to the right of the formula.

$$\begin{pmatrix} 0{,}25 & 0{,}16 \\ 0{,}35 & 0{,}12 \end{pmatrix} \cdot \begin{pmatrix} 237{,}00 \\ 290{,}13 \end{pmatrix} + \begin{pmatrix} 0{,}06 & 0{,}02 \\ 0{,}04 & 0{,}1 \end{pmatrix} \cdot \begin{pmatrix} 16{,}49 \\ 17{,}05 \end{pmatrix} + \begin{pmatrix} 130 \\ 170 \end{pmatrix} =$$

$$= \begin{pmatrix} 105{,}67 \\ 117{,}77 \end{pmatrix} + \begin{pmatrix} 1{,}33 \\ 2{,}36 \end{pmatrix} + \begin{pmatrix} 130 \\ 170 \end{pmatrix} = \begin{pmatrix} 237{,}00 \\ 290{,}13 \end{pmatrix}$$

the second year after the calculations, we get exactly the same result as the total production.

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