ESTIMATE THE SURVIVAL FUNCTION FOR THE NEW MODEL THREE-PARAMETER WEIGHTED NWIKPE DISTRIBUTION

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Abstract

The study seeks to use the theory of weighted distributions to obtain a new proposed probabilistic model known as the Weighted Nwikpe Distribution with three parameters (α, θ, c), as some of its properties have been studied and the survival function was calculated using the method of Maximum likelihood estimation and standard Bayes method Under a quadratic loss function of information either in the experimental side Monte Carlo simulations were employed to generate random data for a sample consisting of five sample sizes (10.25.50.75.100) that follow the Weighted Nwikpe Distribution, with the aim of giving a complete idea of the statistical measures used and using the statistical indicator, mean integral error squares (IMSE) to compare the advantage of The estimation methods used, showed the preference of the standard Bayes method in estimating the survival function over the rest of the other methods for medium and large sample sizes.

As for the practical side, where the distribution was applied to a real sample of (50) observations representing the survival times of people infected with the covid-19 virus, and by means of Goodness of fit of test, the priority of these evidences and their suitability with the new distribution was proven, and the survival function of the real data was estimated using the standard Bayes method. Which appeared advantage in the experimental side.

Keywords: standard Bayes method, Weighted Nwikpe Distribution, Maximum likelihood estimation, quadratic loss function.

1. Introduction

The statistical researcher faces many statistical problems during the process of analyzing the data as well as estimating the parameters related to the distribution, and among those problems that the researcher directs is the process of determining the appropriate and appropriate distribution of the data phenomenon The researchers specialized in the field specialized in the statistical field have worked to develop the probability distributions and move them to the stage of the weighted probability distributions, in order to obtain the best representation of the data with the least errors, especially when the researcher faces the problem of choosing the sample with an equal probability, which made the original distribution limited to the modeling of phenomena It is useless and when it becomes necessary to suggest a specific modification to be done through the weights system in order to obtain vocabulary that has the same appearance of accidents.

recently developed a new continuous probability distribution called the Nwikpe distribution. The Nwikpe distribution is a single parameter distribution derived by taken a two component additive mixture of gamma and exponential distributions. One of the short falls of the Nwikpe distribution is that the distribution has a single parameter. that introducing more parameters into an existing distribution enhances the flexibility of the distribution. Thus, the aim of this paper is to develop a new distribution with two parameters. The new distribution is obtained by taken a three component additive mixture of gamma and exponential distributions, the resultant mixed model is a two parameter probability distribution called the Two-Parameter Nwikpe distribution.

2. Research Problem

The research problem was the emergence of the Corona pandemic (Covid-19), which spread across the continents at a tremendous speed and left hundreds of millions of injuries and tens of millions of deaths in a short period of time, which caused fear and panic in human societies, and hindered the wheel of the global economy, which necessitated research and studies, to explain the behaviour of this pandemic, and try to control it as much as possible.

3. Search Objective

- Using the theory of weighted distributions to construct a new probability Weighted Nwikpe Distribution to address the problem of bias in the evidence as well as to obtain a more flexible distribution.
- Deriving the characteristics of the distribution and estimating the survival function using the greatest possibility method and the standard Bayes method.
- Comparison between the two methods and using the best method to estimate the survival function of people infected with the covid-19 virus.

4. Survival Function[6][8]

One of the methods in statistics is survival analysis, which describes death in living organisms and failures in systems and machines in addition to their uses in the biological and medical aspects. Or death, survival analysis focuses mainly on prediction in determining the probability of risks and is symbolized by the symbol S(t), and it can be expressed mathematically as follows:

$$S(t) = 1 - F(t) \qquad \dots \quad (1)$$

Since:

F(t): Aggregate density function of the random variable t.

T: Represents the time required for failure to occur and is a random variable that represents the time an organism survives to death.

5.TWO Parameter Nwikpe Distribution [10]

The Probability density function of two parameters Nwikpe distribution with parameters θ and α is given by:

$$f(t,\alpha,\theta) = \frac{\theta^3}{(\theta^5+2\alpha+6)}(\theta t^3 + \alpha t + \theta^3)e^{-t\theta}; \ \ t > 0, \alpha > 0, \theta > 0 \qquad \dots \quad (2)$$

α: Shape parameters.

θ : Scale parameter.

and the cumulative distribution function of the two parameters Nwikpe distribution is given by:

$$F(t,\alpha,\theta) = \left[1 - \left(\frac{\theta^3 t^3 + (\alpha+3)(\theta^2 t^2 + 2\theta t)}{(\theta^5+2\alpha+6)}\right)e^{-x\theta}\right]; \ \ t > 0, \alpha > 0, \theta > 0 \qquad \dots \quad (3)$$

The r th moment about origin of TWO Parameter Nwikpe Distribution is given by:

$$\bar{\mu}_r = E(t^r) = \frac{\theta^3}{(\theta^5+2\alpha+6)}\int_0^\infty t^r\,(\theta t^3 + \alpha t + \theta^3)e^{-t\theta}.dt$$

$$\bar{\mu}_r = E(t^r) = \frac{\theta^3[\Gamma(r+4) + \alpha\Gamma(r+3) + \theta^5\Gamma(r+1)]}{\theta^{r+3}(\theta^5+2\alpha+6)}$$

$$\bar{\mu}_1 = \mathrm{E}(\mathrm{t}^1) = \frac{\theta^3[\Gamma(5) + \alpha\Gamma(4) + \theta^5\Gamma(2)]}{\theta^4(\theta^5 + 2\alpha + 6)}$$

$$\bar{\mu}_2 = \mathrm{E}(\mathrm{t}^2) = \frac{\theta^3[\Gamma(6) + \alpha\Gamma(5) + \theta^5\Gamma(3)]}{\theta^5(\theta^5 + 2\alpha + 6)}$$

$$\bar{\mu}_3 = \mathrm{E}(\mathrm{t}^3) = \frac{\theta^3[\Gamma(7) + \alpha\Gamma(6) + \theta^5\Gamma(4)]}{\theta^6(\theta^5 + 2\alpha + 6)}$$

$$\bar{\mu}_4 = \mathrm{E}(\mathrm{t}^4) = \frac{\theta^3[\Gamma(8) + \alpha\Gamma(7) + \theta^5\Gamma(5)]}{\theta^7(\theta^5 + 2\alpha + 6)}$$

Where c=r

$$\bar{\mu}_c = \mathrm{E}(\mathrm{t}^c) = \frac{\theta^3[\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]}{\theta^{c+3}(\theta^5 + 2\alpha + 6)} \qquad \ldots \qquad (4)$$

6- Weighted Three-Parameter Nwikpe Distribution (TPWAND) [1][10][15]

Assume t is a non-negative random variable with probability density function f (t) ,.Let t^c be the non-negative weight function, then, the probability density function of the weighted random variable T w is given by:

$$f_w(x) = \frac{t^c f(t, \alpha, \theta)}{E(t^c)} \qquad \ldots \qquad (5)$$

Where t^cbe the non - negative weight function and $E(t^c) = \int t^c f(t)dt$; 0>∞., we have to obtain the Weighted Three-Parameter Nwikpe Distribution. We have considered the weight function as t^c to obtain the Weighted Three-Parameter Nwikpe Distribution. The probability density function of Weighted Three-Parameter Nwikpe Distribution (TPWAND) is given by:

$$f_w(t) = \frac{\dfrac{\theta^3 t^c(\theta t^3 + \alpha t + \theta^3)e^{-t\theta}}{(\theta^5 + 2\alpha + 6)}}{\dfrac{\theta^3[\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]}{\theta^{c+3}(\theta^5 + 2\alpha + 6)}}$$

$$f_w(t) = \frac{\theta^3 t^c(\theta t^3 + \alpha t + \theta^3)e^{-t\theta}}{(\theta^5 + 2\alpha + 6)} \frac{\theta^{c+3}(\theta^5 + 2\alpha + 6)}{\theta^3[\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]}$$

$$f_w(t) = \frac{\theta^{c+3} t^c(\theta t^3 + \alpha t^2 + \theta^3)e^{-t\theta}}{\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)}; \quad x > 0, \alpha > 0, \theta > 0, c > 0 \qquad \ldots \qquad (6)$$

α: Shape parameters.

θ : Scale parameter.

C: Shape parameters.

Theorem: Three-Parameter Weighted Nwikpe is a Proper PDF This implies that:

$$\int_0^\infty f_w(x)dt = 1$$

$$\int_0^\infty f_w(x)dt = \frac{\theta^{c+3}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\int_0^\infty t^c(\theta t^3+\alpha t^2+\theta^3)e^{-t\theta}dt$$

$$\frac{\theta^{c+3}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)} = z$$

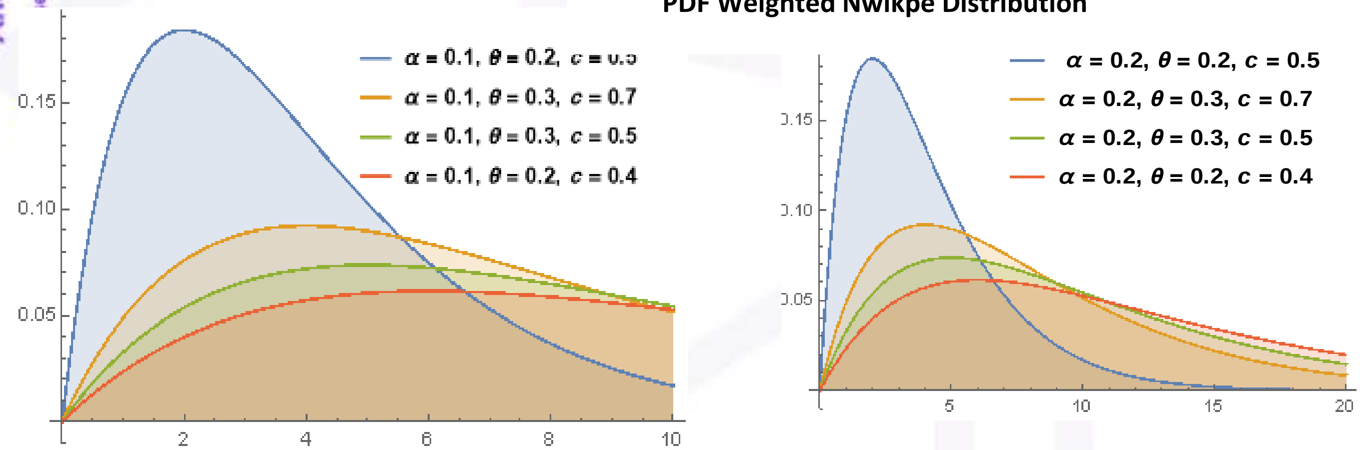
$$\int_0^\infty f_w(x)dt = \theta\int_0^\infty t^{c+3}e^{-t\theta}dt + \alpha\int_0^\infty t^{c+2}e^{-t\theta}dt + \theta^3\int_0^\infty t^c e^{-t\theta}dt$$

$$\int_0^\infty f_w(x)dt = \frac{\theta^c\Gamma(c+4)+\alpha\theta^c\Gamma(c+3)+\theta^{c+5}\Gamma(c+1)}{\theta^c(t^{c+3})}$$

$$= \frac{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}{\theta^{c+3}}\ \frac{\theta^{c+3}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}$$

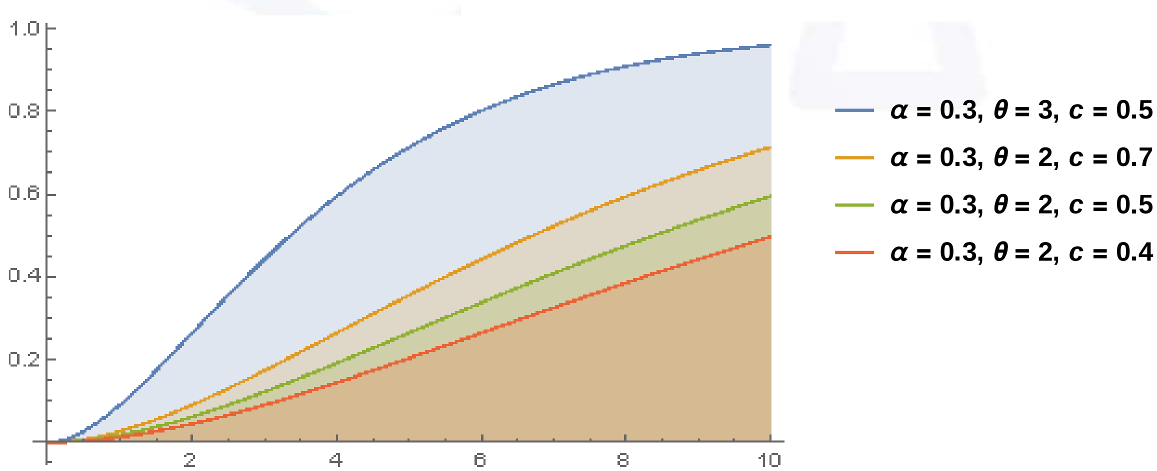
$$= 1$$

PDF Weighted Nwikpe Distribution



The corresponding cumulative distribution function of Weighted Three-Parameter Nwikpe Distribution (TPWAND) is given by:

$$F_w(t) = 1 - \left[\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+ \text{[illegible]}}\right] \quad \ldots \quad (7)$$



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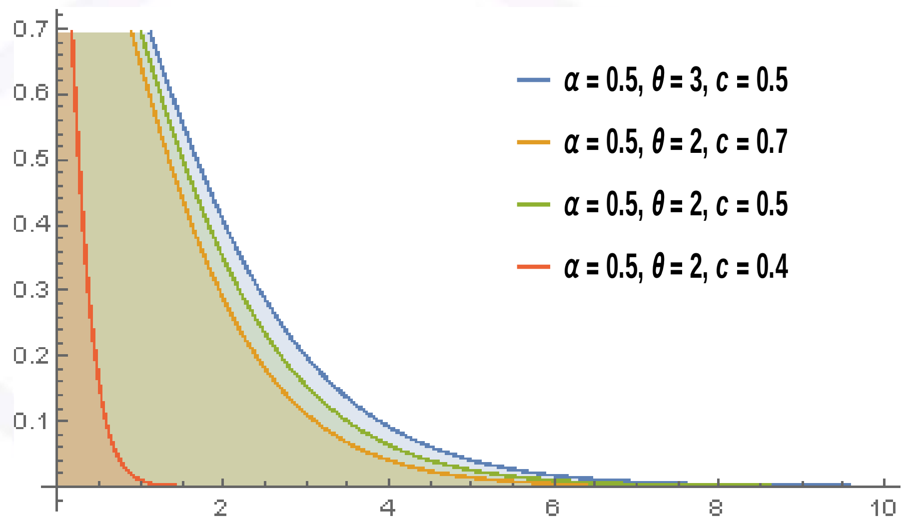
CDF Three-Parameter Weighted Nwikpe

The Survival function of weighted three parameters Weighted Nwikpe is calculated as:

$$S_w(t) = 1 - F_w(t)$$

$$S_w(t) = \frac{\Gamma(c+4,\theta t) + \alpha\Gamma(c+3,\theta t) + \theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)} \quad \ldots \quad (8)$$



Survival function Weighted Nwikpe Distribution

7-moment of Weighted Three-Parameter Nwikpe Distribution

Let Tdenotes the random variable of Weighted Three-Parameter Nwikpe Distribution with Parameters θ, α and c, then the r^{th} of Weighted Three-Parameter Nwikpe Distribution about origin is

$$E(t^r) = \int_0^\infty t^r f_w(t)dt \quad \ldots \quad (9)$$

$$E(t^r) = \int_0^\infty t^r \frac{\theta^{c+3}t^c(\theta t^3 + \alpha t^2 + \theta^3)e^{-t\theta}}{\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)}dt$$

$$E(t^r) = \frac{\theta^{c+3}}{\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)}\int_0^\infty t^{r+c}(\theta t^3 + \alpha t^2 + \theta^3)e^{-t\theta}dt$$

$$\mu_r = E(t^r) = Z\left[\theta\int_0^\infty t^{r+c+3}e^{-t\theta}dt + \alpha\int_0^\infty t^{r+c+2}e^{-t\theta}dt + \theta^3\int_0^\infty t^{r+c}e^{-t\theta}dt\right]$$

$$E(t^r) = Z\left[\theta\int_0^\infty \left(\frac{u}{\theta}\right)^{r+c+3} e^{-u}\frac{du}{\theta} + \alpha\int_0^\infty \left(\frac{u}{\theta}\right)^{r+c+2} e^{-u}\frac{du}{\theta} + \theta^3\int_0^\infty \left(\frac{u}{\theta}\right)^{r+c} e^{-u}\frac{du}{\theta}\right]$$

$$E(t^r) = Z\left[\frac{\Gamma(c+4+r)}{\theta^{r+c+3}} + \frac{\alpha\Gamma(c+3+r)}{\theta^{r+c+3}} + \frac{\theta^2\Gamma(c+1+r)}{\theta^{r+c}}\right]$$

$$E(t^r) = Z\left[\frac{\Gamma(c+4+r)+\alpha\Gamma(c+3+r)}{\theta^{r+c+3}} + \frac{\theta^2\Gamma(c+1+r)}{\theta^{r+c}}\right]$$

$$E(t^r) = Z\left[\frac{\Gamma(c+4+r)+\alpha\Gamma(c+3+r)+\theta^5\Gamma(c+1+r)}{\theta^{r+c+3}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right]$$

$$E(t^r) = \frac{\Gamma(c+4+r)+\alpha\Gamma(c+3+r)+\theta^5\Gamma(c+1+r)}{\theta^{r}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\frac{1}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}$$

$$E(t^r) = \frac{\Gamma(c+4+r)+\alpha\Gamma(c+3+r)+\theta^5\Gamma(c+1+r)}{\theta^{r}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^2} \quad \dots \quad (10)$$

$$\bar{\mu}_1 = E(t) = \frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta^{1}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^2}$$

$$\bar{\mu}_2 = E(t^2) = \frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^{2}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^2}$$

$$\bar{\mu}_3 = E(t^3) = \frac{\Gamma(c+7)+\alpha\Gamma(c+6)+\theta^5\Gamma(c+4)}{\theta^{3}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^2}$$

$$\bar{\mu}_4 = E(t^4) = \frac{\Gamma(c+8)+\alpha\Gamma(c+7)+\theta^5\Gamma(c+5)}{\theta^{4}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^2}$$

8-The Moment Generating Function of the Weighted Three-Parameter Nwikpe Distribution

$$\mu_r = E(t-\mu)^r = \int_0^\infty (t-\mu)^r f_w(t)dt \quad \dots \quad (11)$$

$$E(t-\mu)^r = \int_0^\infty (t-\mu)^r \frac{\theta^{c+3}t^c(\theta t^3+\alpha t^2+\theta^3)e^{-t\theta}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}dt$$

$$\mu_r = E(t-\mu)^r = \frac{\theta^{c+3}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\int_0^\infty (t-\mu)^r t^c(\theta t^3+\alpha t^2+\theta^3)e^{-t\theta}dt$$

$$\mu_r = E(t-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(t)^j(-\mu)^{r-j}$$

$$\mu_r = E(t-\mu)^r = Z\int_0^\infty (t-\mu)^r t^c(\theta t^3+\alpha t^2+\theta^3)e^{-t\theta}dt$$

$$\mu_r = E(t-\mu)^r = Z\int_0^\infty \sum_{j=0}^{r}\binom{r}{j}(t)^j(-\mu)^{r-j}t^c(\theta t^3+\alpha t^2+\theta^3)e^{-t\theta}dt$$

$$\mu_r = E(t-\mu)^r$$
$$= Z\left[\theta\int_0^\infty \sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}t^{j+c+3}\,\mathrm{e}^{-t\theta}dt + \alpha\int_0^\infty \sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}t^{j+c+2}\mathrm{e}^{-t\theta}dt + \theta^3\int_0^\infty \sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}t^{j+c}\mathrm{e}^{-t\theta}dt\right]$$

$$\mu_r = E(t-\mu)^r$$
$$= Z\left[\theta\left(\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\right)\int_0^\infty t^{j+c+3}\mathrm{e}^{-t\theta}dt + \alpha\left(\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\right)\int_0^\infty t^{j+c+2}\mathrm{e}^{-t\theta}dt + \theta^3\left(\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\right)\int_0^\infty t^{j+c3}\mathrm{e}^{-t\theta}dt\right]$$

$$E(t-\mu)^r = Z\left[\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{\Gamma(c+j+4)+\alpha\Gamma(c+j+3)}{\theta^{j+c+3}} + \frac{\theta^2\Gamma(c+j+3)}{\theta^{j+c+1}}\right]$$

$$(t-\mu)^r = Z\left[\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{\Gamma(c+j+4)+\alpha\Gamma(c+j+3)+\theta^5\Gamma(c+j+1)}{\theta^{j+c+3}}\right]$$

$$(t-\mu)^r = \left[\begin{array}{c}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{\Gamma(c+j+4)+\alpha\Gamma(c+j+3)+\theta^5\Gamma(c+j+1)}{\theta^{j+c+3}} \\ \frac{\theta^{c+3}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\end{array}\right]$$

$$(t-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left(\frac{\Gamma(c+j+4)+\alpha\Gamma(c+j+3)+\theta^5\Gamma(c+j+1)}{\theta^j[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right)$$

$$E(T^j) = \frac{\Gamma(c+j+4)+\alpha\Gamma(c+j+3)+\theta^5\Gamma(c+j+1)}{\theta^j[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]} \quad \ldots \quad (12)$$

The Second Moments about the Mean of the Weighted Three-Parameter Nwikpe Distribution

$$\mu_2 = (t-\mu)^2 = \sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}(-\mu^j)$$
$$= \sum_{j=0}^{2}\binom{2}{j}(-ET)^{2-j}(-ET^j)$$
$$= \binom{2}{0}(-ET)^2(-ET^0) + \binom{2}{1}(-ET)^1(-ET^1) + \binom{2}{2}(-ET)^0(-ET^2)$$
$$= (ET^2) - (ET)^2$$
$$(t-\mu)^2 = \frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\cdot\left(\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right)^2$$

$$\sigma^2 = \frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\cdot\left(\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right)^2 \quad \ldots \quad (13)$$

$$\sigma = \sqrt{\frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}\,\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]} \quad \ldots \quad (14)$$

The Third Moments about the Mean of the Weighted Three-Parameter Nwikpe Distribution

$$\mu_3 = (t - \mu)^3 = (ET^3) - 3ETET^2 + 2(ET)^3$$

$$= \frac{\Gamma(c+7)+\alpha\Gamma(c+6)+\theta^5\Gamma(c+4)}{\theta^3[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]} - 3\left[\frac{\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}{\frac{\Gamma(c+6)+\alpha\Gamma(c+6)+\theta^5\Gamma(c+6)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}\right] + 2\left[\frac{\Gamma(c+5)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right]^3$$

The four Moments about the Mean of the Weighted Three-Parameter Nwikpe Distribution

$$\mu_4 = (t - \mu)^4 = ET^4 - 4ET^3ET + 6ET^2(ET)^2 - 3(ET)$$

$$\mu_4 = \frac{\Gamma(c+8)+\alpha\Gamma(c+7)+\theta^5\Gamma(c+5)}{\theta^4[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]} - 4\left[\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\frac{\Gamma(c+7)+\alpha\Gamma(c+6)+\theta^5\Gamma(c+4)}{\theta^3[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right] + 6\left[\frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\left(\frac{\Gamma(c+j+4)+\alpha\Gamma(c+j+3)+\theta^5\Gamma(c+j+1)}{\theta^j[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right)^2\right] - 3\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}$$

9-Coefficient of Variation of the Weighted Three-Parameter Nwikpe Distribution

The coefficient of variation of a random variable T, is given by

$$\mathrm{C.V} = \frac{\sigma}{\mu} \times 100\%$$

$$\mathrm{C.V} = \frac{\sqrt{\frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}\;\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}{\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}} \quad \dots \quad (15)$$

10-Coefficient of Skewness of the Weighted Three-Parameter Nwikpe Distribution

If T follows the Weighted Three-Parameter Nwikpe Distribution its coefficient of skewness is computed as follows:

$$\boldsymbol{S_K = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}}$$

$$\boldsymbol{S_K} = \left[\begin{array}{l}\frac{\Gamma(c+7)+\alpha\Gamma(c+6)+\theta^5\Gamma(c+4)}{\theta^3[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]} \\ -3\left(\frac{\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}{\frac{\Gamma(c+6)+\alpha\Gamma(c+6)+\theta^5\Gamma(c+6)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}\right) \\ +2\left[\frac{\Gamma(c+5)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}\right]^3\end{array}\right] \div \left[\frac{\sqrt{\frac{\Gamma(c+6)+\alpha\Gamma(c+5)+\theta^5\Gamma(c+3)}{\theta^2[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}}{\frac{\Gamma(c+5)+\alpha\Gamma(c+4)+\theta^5\Gamma(c+2)}{\theta\,[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]}}\right]^{\frac{3}{2}} \quad \dots \quad \mathbf{(16)}$$

11.Maximum Likelihood Method Estimation[12][13]

The ML method is considered one of the most important traditional estimation methods in the estimation process and the most widely used because it has good characteristics, including sufficiency, consistency, non-bias, and has the least variance, and it is more accurate when the sample size is large, and the first to formulate this method is the scientist (CFGauss) It was applied for the first time by the researcher (SAFisher) in the year (1922), and the estimators extracted according to the method of Maximum Likelihood Method Estimator are characterized by having some characteristics of a good estimator, and that the principle and goal of this method is to find estimation values for the parameters that we want to estimate, and that By making the possible function at its maximum limit.

$$Lf(x_1.x_2.\dots.x_n)=\prod_{i=1}^{n} f(x.\theta.\alpha.c) \qquad \dots (17)$$

$$=\prod_{i=1}^{n}\left[\frac{\theta^{c+3}t^{c}(\theta t^{3}+\alpha t^{2}+\theta^{3})e^{-t\theta}}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^{5}\Gamma(c+1)}\right]$$

$$=\frac{\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}}{[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^{5}\Gamma(c+1)]^{n}}\prod_{i=1}^{n}(\theta t^{3}+\alpha t^{2}+\theta^{3})$$

$$=\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^{5}\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^{3}+\alpha t^{2}+\theta^{3})$$

$$\ln Lf(x_1.x_2.\dots.x_n)=3n\,ln\theta+n(ln\theta)-\theta\sum_{i=1}^{n} ti-nln[(c+4)+\alpha\Gamma(c+3)+\theta^{5}\Gamma(c+1)]+\sum_{i=1}^{n} ln(\theta t^{3}+\alpha t^{2}+\theta^{3})$$

i = 1,2, … n

$$\frac{dlnL}{d\theta}=\frac{3n}{\theta}-\frac{n}{\theta}+\sum_{i=1}^{n} ti+5\theta^{4}\Gamma(c+1)+3\theta\sum_{i=1}^{n} ti^{2} \qquad \dots (18)$$

$$\frac{dlnL}{d\alpha}=\Gamma(c+3)+\sum_{i=1}^{n} t^{2}i \qquad \dots (19)$$

$$\frac{dlnL}{dc}=\alpha\acute{\Gamma}(c+3)+\theta^{5}\acute{\Gamma}(c+1)-\frac{n}{c} \qquad \dots (20)$$

$$\acute{\Gamma}(c+3)=\psi(c+3)\,\Gamma(c+3)$$

$$\psi(z)=\frac{dln(z)}{dz}=\frac{\acute{\Gamma}(z)}{\Gamma(z)}$$

12. Standard Informative Bayesian Estimator[4][5]

The Bayesian method or the Bayes method is named after Thomas Bayes, as there are two schools in the first estimation called the classical school, which assumes that parameters are fixed quantities that are estimated by classical

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methods such as the method of greatest possibility, method of least squares, moment method and others, and the other is called the Bayesian school Which assumes that the parameters are random variables with a probability distribution called the priority distribution, and this distribution is usually inappropriate, meaning that its integration for each field is not equal to one and is recognized by previous experiments or from data or by the theory that governs This phenomenon, and the data of the current sample has a role in the estimation process where the possibility function is combined from the data of the current sample with the prior distribution using the inverse Bayes formula To get the Posterior distribution. The Bayes method differs from the classical methods in that in the Bayes method, a decision is made to either reduce the loss or maximize the benefit.

The standard Bayes estimator depends on the suffix distribution function which includes the previous information about the parameter (θ) and sample observations of the current$(x_1.x_2.\dots.x_n)$Under the Bayes theorem, the suffix distribution of the parameter (θ) can be obtained by using The Inverse Bayesian formula is as follows:

$$h\left(\underline{\theta}|x_1.x_2.\dots.x_n\right) = \frac{\pi(\underline{\theta})\prod_{i=1}^{n} f(x_1.x_2.\dots.x_n|\underline{\theta})}{\int_{\forall\underline{\theta}} \pi(\underline{\theta})\prod_{i=1}^{n} f(x_1.x_2.\dots.x_n|\underline{\theta})d\underline{\theta}} \quad \dots(21)$$

whereas:

$\pi(\underline{\theta})$: The initial distribution of the parameter.

$f(x_1.x_2.\dots.x_n|\underline{\theta})$: Possibility function for sample observations of size n.

$h\left(\underline{\theta}|x_1.x_2.\dots.x_n\right)$: Post − distribution of parameters θ.

In order to find the standard Bayesian estimator, one of the Loss Functions must be used. In this message, the Squared Error Loss function will be used to find the informational standard Bayesian estimator, as follows:

12.1. Standard Bayesian Estimator under Squared Error Loss function [4][6]

The squared loss function, which is also called the squared error loss function, is a symmetrical loss function, that is, the amount of loss in the loss function for positive error is equal to the amount of loss for negative error and in the same direction. The loss function is as small as possible which is always acceptable. The quadratic loss function is defined by the following formula:

$$L_1(\underline{\hat{\theta}}.\underline{\theta}) = a_0(\underline{\hat{\theta}} - \underline{\theta})^2 \qquad ...(22)$$

Therefore, the Bayesian estimator under the Squared Error Loss function, which makes the risk function as little as possible, which represents the expectation of the loss function after finding the first derivative with respect to the parameter to be estimated and equalizing it to zero, we get:

$$L_1(\underline{\hat{\theta}}.\underline{\theta}) = (\underline{\hat{\theta}} - \underline{\theta})^2$$

$$\text{Bayes Risk} = E(\underline{\hat{\theta}} - \underline{\theta})^2)$$

$$E(\underline{\hat{\theta}} - \underline{\theta})^2 = \int_{\forall\underline{\theta}} (\underline{\hat{\theta}} - \underline{\theta})^2 h\,(\underline{\theta}|\underline{x})d\underline{\theta}$$

$$E(\underline{\hat{\theta}} - \underline{\theta})^2 = \int_{\forall\underline{\theta}} (\underline{\hat{\theta}}^2 - 2\underline{\theta}\,\underline{\hat{\theta}} + \underline{\theta}^2)h\,(\underline{\theta}|\underline{x})d\underline{\theta}$$

$$E\,(\underline{\hat{\theta}} - \underline{\theta})^2 = \underline{\hat{\theta}}^2 - 2\underline{\hat{\theta}}E(\underline{\theta}|\underline{x}) + E(\underline{\theta}^2|\underline{x}) \qquad ...(23)$$

By partially differentiating equation (23) with respect to $(\hat{\theta})$ and setting the derivative equal to zero, we get:

$$\therefore \underline{\hat{\theta}}_{SBSEL} = E(\underline{\theta}|\underline{x}) \qquad ...(24)$$

Therefore, the Bayes estimator under the Squared Error Loss function, after taking the root of the force q for both sides of equation (24), we get the following:

$$\hat{S}_{SBSEL}(x_1.x_2.\dots.x_n) = E(S|x_1.x_2.\dots.x_n) \qquad ...(25)$$

Now we need to give the initial distributions of the parameters to be estimated θ, α, β, and according to what information is available to the researcher about the initial distributions of the parameters, suppose that the initial distributions of those parameters will be as follows:

$\theta \sim \text{Gamma}\ (a_1, b_1)$

$\alpha \sim \text{Gamma}\ (a_2, b_2$

$c \sim \text{Exp}\ (\delta)$

Thus, the priority distribution function for each parameter is formed as follows:

$$\pi_1(\theta) \propto \frac{{b_1}^{a_1}}{\Gamma(a_1)} \theta^{a_1-1} e^{-b1\theta} \quad ;\theta > 0 \qquad ...(26)$$

$$\pi_2(\alpha) \propto \frac{{b_2}^{a_2}}{\Gamma(a_2)} \alpha^{a_2-1} e^{-b_2\alpha} \quad ;\alpha > 0 \qquad ...(27)$$

$$\pi_3(c) \propto ;\ ce^{-c\delta} \quad ;\delta\ > 0 \qquad ...(28)$$

Therefore, the joint priority is as follows:

$$\pi(\theta, \alpha, c) \propto \frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} \theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ ce^{-\delta c} \qquad ...(29)$$

Refer to the researcher (HAN) [22] The parameters $(a_1, a_2, b_1, b_2, \delta)$ can be selected in the form that ensures that the preliminary probability density is $\pi_3(c), \pi_2(\alpha), \pi_1(\theta)$ annexed to the basic parameters to be appreciated and since the first derivation of primary probability density is:

$$\frac{d\pi_1(\theta)}{d\theta} = \frac{{b_1}^{a_1}}{\Gamma(a_1)}\theta^{a_1-2}e^{-b1\theta}\ \left(-b_1\theta + (a_1 - 1)\right) \quad \dots (30)$$

$$\frac{d\pi_2(\alpha)}{d\alpha} = \frac{{b_1}^{a_2}}{\Gamma(a_1)}\theta^{a_2-2}e^{-b2\theta}\ \left(-b2\theta + (a_2 - 1)\right) \quad \dots (31)$$

$$\frac{d\pi_3(\beta)}{dc} = \delta(\delta c - 2)e^{-\delta c} \quad \dots (32)$$

The researcher has proven ^[12] (Berger, 1985) that large basic parameter values reduce the accuracy of Bayesian estimations, so these values must be chosen within the range $0 < \theta, \alpha, c < C$, where C is a real constant that is chosen in a way that does not go far from The values of $(a_1, a_2, \delta, b_1, b_2)$ are close to the parameters (θ, α, c) in order to preserve the immunity of Bayesian estimators.

Press (2001) suggested using very small positive values for parameters in the initial distribution, so we will assume that:

$$a_1 = b_1 = a_2 = b_2 = \delta = 0.1e^{-11}$$

The marginal density function of the sample observations is in the following form:

$$f(x_1.x_2.\dots.x_n) = \int_\theta\int_\alpha\int_c \pi(\theta.\alpha.c)\ L\ d\theta d\alpha d\beta \quad \dots (33)$$

$$\pi(\theta,\alpha,c)\ L \propto \pi(\theta,\alpha,c)\ L \propto \theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}[\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)\right) \quad \dots (34)$$

$$A = \frac{\delta {b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}$$

According to Bayes' theorem, the posterior distribution function (Posterior pdf) for the parameters (θ, α, c) can be obtained by using the inverse Bayes formula by dividing the joint distribution function by the marginal density function, as follows:

$$h\,(\theta,\alpha,c|x_1.x_2.\dots.x_n) = \frac{f(\theta,\alpha,c,X_1,\dots,X_n)}{\iiint_{\theta,\alpha,c} f(\theta,\alpha,\beta,X_1,\dots,X_n)d\theta d\alpha dc}$$

$$= \frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}[\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)\right)}{\int_\theta\int_\alpha\int_c \theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}[\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)\right)d\theta d\alpha dc} \dots (35)$$

By using Squared Error Loss function, the standard Bayes estimator of the survival function can be obtained, as follows:

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$$\hat{S}_{SBSEL}(x_1.x_2.\dots.x_n) = E(S|x_1.x_2.\dots.x_n) \quad \dots (36)$$

$$= \int_\theta\int_\alpha\int_c S(x)h\,(\theta,\alpha,C|x_1.x_2.\dots.x_n)d\theta d\alpha dc$$

$$= \int_\theta\int_\alpha\int_c \left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)} \frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n} ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc \dots (37)$$

We note that equation (37) represents a non-linear equation system that is not theoretically tight and cannot be solved by ordinary analytical methods. Therefore, an approximate method must be used to calculate these complex integrals. Therefore, Lindely Approximation will be used to find the standard Bayesian estimator for the survival function. under the general entropy function.

13.Lindely Approximation [14[16]

The researcher (Lindley) in the year (1980) put an approximate solution to the integration resulting from the use of the Bayesian estimator method.

$$E[u(\underline{\theta})\backslash x] = \frac{\int_\Omega u(\underline{\theta})e^{L(\underline{\theta})+\rho(\underline{\theta})}d\underline{\theta}}{\int_\Omega e^{L(\underline{\theta})+\rho(\underline{\theta})}d\underline{\theta}} \quad \dots (38)$$

Since:

$L(\underline{\theta})$: The logarithm of the maximum possibility function. $\rho(\underline{\theta})$:
The logarithm of the previous distribution function for the parameter (θ) .

$u(\underline{\theta})$: Any function of the parameter (θ).

The researcher proposed to Lindley the following formula for solving the integrals resulting from the Bayes formula, as follows:

$$E[u(\underline{\theta})\,/\,x] = u(\hat{\theta}) + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}[u_{ij}+2u_i\rho_j]\,\sigma_{ij} + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}L_{ijk}u_l\sigma_{ij}\sigma_{kl} \dots (39)$$

Since:

m**:** represents the number of parameters, (m=3)**.**

$u(\hat{\theta})$: Estimated maximum survival function.

$$L_{ijk} = \frac{\partial^3 L(\underline{\theta})}{\partial\underline{\theta}_1\,\partial\underline{\theta}_2\,\partial\underline{\theta}_3}\Big|_{\underline{\theta}=\underline{\hat{\theta}}} \qquad i.j.k = 1.2.3 \quad \dots (40)$$

$\partial\underline{\theta}_1 = \partial\theta$

$\partial\underline{\theta}_2 = \partial\alpha$

$\partial\underline{\theta}_3 = \partial\beta$

$$\sigma_{ij} = -\left(\frac{\partial^2 L(\underline{\theta})}{\partial\underline{\theta}_i\,\partial\underline{\theta}_j}\Big|_{\underline{\theta}=\underline{\hat{\theta}}}\right)^{-1} \quad \theta_1=\theta;\theta_2=\alpha;\theta_3=c\ ,i.j=1,2,3 \quad \dots (41)$$

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$$\rho = \text{Log}\left(\pi(\underline{\theta})\right)$$

$$= \rho = \text{Log}\left(\pi(\underline{\theta})\right) = \text{Log}\left(\frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ ce^{-\delta c}\right) \quad \ldots (42)$$

$$\rho_i = \frac{\partial \log(\rho)}{\partial \underline{\theta}_i} \qquad i = 1,2,3 \quad \ldots (43)$$

$$u_i = \frac{\partial u(\underline{\theta})}{\partial \underline{\theta}_i}; \qquad i = 1,2,3 \quad \ldots (44)$$

$$u_{ij} = \frac{\partial u^2}{\partial \underline{\theta}_i\, \partial \underline{\theta}_j} \qquad i.j = 1,2,3 \quad \ldots (45)$$

And to get the standard Bayes estimator for the survival function under a general entropy function and assuming that:

$$(\theta;\alpha;\beta) = \int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.$$

$$\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc \ldots (46)$$

by deriving the equation (46):

$$u_1 = \frac{du\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{d\theta}$$
$$\frac{\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc}{}$$

$$u_2 = \frac{du\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{\partial\alpha}$$
$$\frac{\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc}{}$$

$$u_3 = \frac{du\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{\partial c}$$
$$\frac{\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc}{}$$

$$u_{12} = \frac{du\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{\partial\theta\,\partial\alpha}$$
$$\frac{\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc}{}$$

$$u_{21} = u_{12}$$

$$u_{22} = \frac{\partial^2 u\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{(\partial\alpha)^2}$$
$$\frac{\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\ e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc}{}$$

$$u_{13} = \frac{\partial^2 u\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{\partial\theta\,\partial c}$$

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$$\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc$$

$$u_{13}=u_{31}$$

$$u_{13}=\frac{\partial^2 u\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{\partial\alpha\,\partial c}$$

$$\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc$$

$$u_{32}=u_{23}$$

$$u_{33}=\frac{\partial^2 u\int_\theta\int_\alpha\int_c\left(\frac{\Gamma(c+4,\theta t)+\alpha\Gamma(c+3,\theta t)+\theta^5\Gamma(c+1,\theta t)}{\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)}\right.}{(\partial c)^2}$$

$$\left.\frac{\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)}{\int_\theta\int_\alpha\int_c\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,e^{-\delta t}\left(\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)\right)d\theta d\alpha dc}\right)d\theta d\alpha dc$$

$$L_{123}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\theta\,\partial\alpha\,\partial c}$$

$$L_{132}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\theta\,\partial c\,\partial\alpha}$$

$$L_{213}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\alpha\,\partial\theta\,\partial c}$$

$$L_{231}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\alpha\,\partial c\,\partial\theta}$$

$$L_{312}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial\theta\,\partial\alpha}$$

$$L_{321}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial\alpha\,\partial\theta}$$

$$L_{112}=\frac{\partial^3\,\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\theta\,\partial\theta\,\partial\alpha}$$

$$L_{332}=\frac{\partial^3\,\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\theta\,\partial\theta\,\partial c}$$

$$L_{323}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial\alpha\,\partial c}$$

$$L_{233}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\alpha\,\partial c\,\partial c}$$

$$L_{113}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\theta\,\partial\theta\,\partial c}$$

$$L_{131}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\theta\,\partial c\,\partial\theta}$$

$$L_{311}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial\theta\,\partial\theta}$$

$$L_{233}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial\alpha\,\partial c\,\partial c}$$

$$L_{323}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial\alpha\,\partial c}$$

$$L_{322}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial\alpha\,\partial\alpha}$$

$$L_{333}=\frac{\partial^3\theta^{n(c+3)}e^{-\sum_{i=1}^{n}ti}[\Gamma(c+4)+\alpha\Gamma(c+3)+\theta^5\Gamma(c+1)]^{-n}\prod_{i=1}^{n}(\theta t^3+\alpha t^2+\theta^3)}{\partial c\,\partial c\,\partial c}$$

$$\rho_1=\frac{\partial \mathrm{Log}\left(\frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,ce^{-\delta c}\right)}{\partial\theta}$$

$$\rho_2=\frac{\partial \mathrm{Log}\left(\frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\theta^{a_1-1}\alpha^{a_2-1}e^{-b_1\theta}e^{-b_2\alpha}\,ce^{-\delta c}\right)}{\partial\alpha}$$

$$\rho_2 = \frac{\partial \mathrm{Log}\left(\frac{b_1{}^{a_1} b_2{}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} \theta^{a_1-1} \alpha^{a_2-1} e^{-b_1\theta} e^{-b_2\alpha}\, c e^{-\delta c}\right)}{\partial c}$$

$$\sigma_{11} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial\theta\, \partial\theta}\right)^{-1}$$

$$\sigma_{12} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial\theta\, \partial\alpha}\right)^{-1}$$

$$\sigma_{13} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial\theta\, \partial c}\right)^{-1}$$

$$\sigma_{21} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial c\, \partial\theta}\right)^{-1}$$

$$\sigma_{22} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial\alpha\, \partial\alpha}\right)^{-1}$$

$$\sigma_{23} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial\alpha\, \partial c}\right)^{-1}$$

$$\sigma_{32} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial c\, \partial\alpha}\right)^{-1}$$

$$\sigma_{31} = -\left(\frac{\partial^2\, \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial c\, \partial\theta}\right)^{-1}$$

$$\sigma_{33} = -\left(\frac{\partial^2 = \theta^{n(c+3)} e^{-\sum_{i=1}^{n} ti} [\Gamma(c+4) + \alpha\Gamma(c+3) + \theta^5\Gamma(c+1)]^{-n} \prod_{i=1}^{n}(\theta t^3 + \alpha t^2 + \theta^3)}{\partial c\, \partial c}\right)^{-1}$$

Equation (39) is as follows:

$$\begin{aligned}\hat{S}_{SBSEL} = \hat{S}_{mle} &+ u_1\rho_1\sigma_{11} + u_1\rho_2\sigma_{12} + +u_1\rho_3\sigma_{13} + 0.5L_{231}u_1\sigma_{23}\sigma_{11} + 0.5L_{233}u_1\sigma_{23}\sigma_{31} + 0.5L_{311}u_1\sigma_{31}\sigma_{11} \\ &+ 0.5L_{312}u_1\sigma_{31}\sigma_{21} + 0.5L_{313}u_1\sigma_{31}^2 + 0.5L_{321}u_1\sigma_{32}\sigma_{11} + 0.5L_{323}u_1\sigma_{32}\sigma_{31} + 0.5L_{331}u_1\sigma_{33}\sigma_{13} \\ &+ 0.5L_{332}u_1\sigma_{33}\sigma_{21} + 0.5L_{333}u_1\sigma_{33}\sigma_{31} + 0.5L_{111}u_1\sigma_{11}^2 + 0.5L_{112}u_1\sigma_{11}\sigma_{21} \\ &+ 0.5L_{113}u_1\sigma_{11}\sigma_{31} \qquad \ldots (47)\end{aligned}$$

Note that all the integrals and derivations of the aforementioned equations were made within the functions of the Matlab program because it is difficult to solve them manually.

14-Simulations by Monte-Carlo method[2][7]

In order to compare the efficiency of the Bayesian estimation method and the Bayesian prediction method to obtain good estimates of survival function with the desired characteristics, the simulation method was employed by the (Monte-Carlo) method (10.25.50.75.100), noting that the repetition of the experiment was (1000) where the number of estimated experiments was three experiments applied with the MATLAB program)) and the following is a detailed presentation of the experiments.

Table (1) default values for distribution parameters

Experiment	θ	α	β
1	2	3	3
2	2	3.5	2

Table (2) the real and estimated values of the survival function by all estimation methods and the mean values of integral error squares (IMSE) at each sample size

		Model 1	θ=2 α=3	β=3			Model 2	θ=2 α=3.5	β=2
n	**ti**	Real(R(t))	$\hat{S}(t)_{Bayes}$	$\hat{S}(t)_{ML}$	**n**	**ti**	Real(R(t))	$\hat{S}(t)_{Bayes}$	$\hat{S}(t)_{ML}$
	0.1	0.99986	0.95000	0.97500		0.1	0.99990	0.95000	0.97500
	0.2	0.99922	0.90000	0.92500		0.2	0.99944	0.90000	0.92500
	0.3	0.99785	0.85000	0.87500		0.3	0.99846	0.85000	0.87500
	0.4	0.99559	0.80000	0.82500		0.4	0.99684	0.80000	0.82500
	0.5	0.99232	0.75000	0.77500	10	0.5	0.99449	0.75000	0.77500
10	0.6	0.98791	0.70000	0.72500		0.6	0.99132	0.70000	0.72500
	0.7	0.98227	0.65000	0.67500		0.7	0.98727	0.65000	0.67500
	0.8	0.97533	0.60000	0.62500		0.8	0.98227	0.60000	0.62500
	0.9	0.96703	0.55000	0.57500		0.9	0.97627	0.55000	0.57500
	1	0.95730	0.50000	0.52500		1	0.96923	0.50000	0.52500
		IMSE	0.084837	0.052439			IMSE	0.087958	0.065356
		Best	$\hat{S}(t)_{ML}$				Best	$\hat{S}(t)_{ML}$	
	0.1	0.92265	0.97500	0.99922		0.1	0.99990	0.97500	0.98750
	0.2	0.83380	0.95000	0.99736		0.2	0.99944	0.95000	0.96250
	0.3	0.81557	0.92500	0.99445		0.3	0.99846	0.92500	0.93750
	0.4	0.78797	0.90000	0.99044		0.4	0.99684	0.90000	0.91250
	0.5	0.68657	0.87500	0.98531	25	0.5	0.99449	0.87500	0.88750
25	0.6	0.44426	0.85000	0.97902		0.6	0.99132	0.85000	0.86250
	0.7	0.44221	0.82500	0.97153		0.7	0.98727	0.82500	0.83750
	0.8	0.37750	0.80000	0.96282		0.8	0.98227	0.80000	0.81250
	0.9	0.29328	0.77500	0.95288		0.9	0.97627	0.77500	0.78750
	1	0.08620	0.75000	0.94170		1	0.96923	0.75000	0.76250
		IMSE	0.018559	0.005641	IMSE			0.020022	0.007002
		Best	$\hat{S}(t)_{ML}$				Best	$\hat{S}(t)_{ML}$	
	0.1	0.99986	0.92948	0.99167		0.1	0.99990	0.99981	0.95238
	0.2	0.99922	0.89808	0.97500		0.2	0.99944	0.99912	0.90476
	0.3	0.99785	0.89572	0.95833		0.3	0.99846	0.99782	0.85714
	0.4	0.99559	0.79236	0.94167		0.4	0.99684	0.99582	0.80952
	0.5	0.99232	0.68792	0.92500	50	0.5	0.99449	0.99304	0.76190
50	0.6	0.98791	0.46235	0.90833		0.6	0.99132	0.98942	0.71429
	0.7	0.98227	0.44561	0.89167		0.7	0.98727	0.98491	0.66667
	0.8	0.97533	0.36764	0.87500		0.8	0.98227	0.97946	0.61905
	0.9	0.96703	0.28842	0.85833		0.9	0.97627	0.97302	0.57143
	1	0.95730	0.08791	0.84167		1	0.96923	0.96557	0.52381
		IMSE	0.000037	0.005945	IMSE			0.000085	0.006787
		Best	$\hat{S}(t)$**Bayes**				Best	$\hat{S}(t)_{Bayes}$	
	0.1	0.99986	0.99978	0.95000		0.1	0.99990	0.99983	0.99375
	0.2	0.99922	0.99895	0.90000		0.2	0.99944	0.99918	0.98125
	0.3	0.99785	0.99732	0.85000		0.3	0.99846	0.99792	0.96875
75	0.4	0.99559	0.99475	0.80000		0.4	0.99684	0.99597	0.95625
	0.5	0.99232	0.99114	0.75000	75	0.5	0.99449	0.99324	0.94375
	0.6	0.98791	0.98640	0.70000		0.6	0.99132	0.98967	0.93125
	0.7	0.98227	0.98045	0.65000		0.7	0.98727	0.98521	0.91875
	0.8	0.97533	0.97322	0.60000		0.8	0.98227	0.97979	0.90625

	0.9	0.96703	0.96466	0.55000		0.9	0.97627	0.97339	0.89375
	1	0.95730	0.95473	0.50000		1	0.96923	0.96597	0.88125
		IMSE	0.000036	0.002828	IMSE			0.000045	0.003411
		Best	$\hat{S}(t)_{Bayes}$				Best	$\hat{S}(t)_{Bayes}$	
100	0.1	0.99986	0.95000	0.99500	100	0.1	0.99990	0.99984	0.99500
	0.2	0.99922	0.92500	0.98500		0.2	0.99944	0.99920	0.98500
	0.3	0.99785	0.90000	0.97500		0.3	0.99846	0.99797	0.97500
	0.4	0.99559	0.87500	0.96500		0.4	0.99684	0.99604	0.96500
	0.5	0.99232	0.85000	0.95500		0.5	0.99449	0.99334	0.95500
	0.6	0.98791	0.82500	0.94500		0.6	0.99132	0.98980	0.94500
	0.7	0.98227	0.80000	0.93500		0.7	0.98727	0.98535	0.93500
	0.8	0.97533	0.77500	0.92500		0.8	0.98227	0.97995	0.92500
	0.9	0.96703	0.75000	0.91500		0.9	0.97627	0.97356	0.91500
	1	0.95730	0.95000	0.90500		1	0.96923	0.96614	0.90500
		IMSE	0.000139	0.001513	IMSE			0.000053	0.001939
		Best	$\hat{S}(t)_{Bayes}$				Best	$\hat{S}(t)_{Bayes}$	

15-Applied side[9][11]

The data related to the message for a number of people infected with the new COVD-19 virus were collected from the records of the Department of Al-Hussein Teaching Hospital in the Holy Karbala Governorate, Fever Department. The number is (50) views, representing the times of patients' stay in days under observation and treatment until fulfillment. The data was tabulated for the infected persons for the purpose of obtaining Survival time by subtracting the date of infection with the virus from the date of death, as follows:

Table (3) Represents the times of stay for patients with covid-19

0,01	**0,11**	**0,16**	**0,25**	**0,45**
0,05	**0,14**	**0,17**	**0,29**	**0,57**
0,03	**0,13**	**0,18**	**0,29**	**0,59**
0,04	**0,12**	**0,20**	**0,30**	**0,63**
0,01	**0,13**	**0,20**	**0,34**	**0,71**
0,04	**0,13**	**0,21**	**0,41**	**0,72**
0,03	**0,11**	**0,22**	**0,41**	**0,80**
0,07	**0,14**	**0,22**	**0,42**	**0,89**
0,04	**0,2**	**0,22**	**0,43**	**1,00**
0,09	**0,15**	**0,24**	**0,49**	**1,10**

Table (4) Results of the good-nest of fit test

Distribution	χ_c^2	χ_t^2	**Sig,**	**Decision**
Weighted Three-Parameter Nwikpe Distribution	**3,22**	**3,84**	**,0627**	**Not RejectH_0**

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We note from Table (4) that the value of χ_c^2calculated according is greater than the tabular value, so we do not reject the null hypothesis that the data are distributed according to Nwikpe Distribution Weighted.

Table (**5**) shows the criteria for the trade-off between the fret distribution, the gamma distribution, and the proposed distribution in representing the real data.

Distribution	AIC	AICc	BIC
Gama	3,5818	5,1152	8,2573
Frecht	0,7434	4,3771	7,5192
Nwikpe Distribution	0, 2438	0,5437	0,8932
Nwikpe Distribution Weighted	0, 1631	0,43331	0,7232

We note that the best distribution is Nwikpe Distribution Weighted because it has the lowest value of the three criteria.

Table (6) Weighted Least Squares Estimates of Survival Function for Real Data

		$\hat{S}(t)_{Bayes}$			$\hat{S}(t)_{Bayes}$			$\hat{S}(t)_{Bayes}$
1	0,01	0,95778	10	**0,11**	0,72660	19	0,16	0,61315
2	0,05	0,92657	11	0,14	0,65976	20	0,17	0,60607
3	0,03	0,884744	12	0,13	0,65649	21	0,18	0,57332
4	0,04	0,88323	13	0,12	0,65599	22	0,20	0,55668
5	0,01	0,882338	14	0,13	0,64472	23	0,20	0,54312
6	0,04	0,87384	15	0,13	0,64352	24	0,21	0,50146
7	0,03	0,80165	16	0,11	0,64011	25	0,22	0,49879
8	0,07	0,78818	17	0,14	0,62696	26	0,22	0,48454
9	0,04	0,76191	18	0,2	0,62006	27	0,22	0,47500
i	t	$\hat{S}(t)_{ML}$	i	t	$\hat{S}(t)_{ML}$	i	t	$\hat{S}(t)_{ML}$
27	0,24	0,47500	37	0,49	0,24185	47	0,25	0,06239
28	0,25	0,47389	38	0,45	0,23517	48	0,29	0,04409
29	0,29	0,46889	39	0,57	0,22884	49	0,29	0,02938
30	0,29	0,43766	40	0,59	0,18397	50	0,25	0,02116
31	0,30	0,42448	41	0,63	0,18309			
32	0,34	0,37026	42	0,71	0,14989			
33	0,41	0,36767	43	0,72	0,13665			
34	0,41	0,35359	44	0,80	0,12662			
35	0,42	0,30730	45	0,89	0,08425			
36	0,43	0,24488	46	1,00	0,08163			

9-Conclusions

- The results of the simulation experiments showed that standard Pisa is the best for estimating the survival function for medium and large sample sizes.

- The values of the IMSE statistical scale decrease as the sample size increases, and this matches the statistical theory of this indicator.
- The values of the survival function decreased with the increase in time (t), and this corresponds to what was presented in the theoretical side about the behavior of this function.
- Interest in obtaining data on corona virus disease in all governorates of Iraq to calculate the survival function and the risk function.
- Corona virus disease is one of the highly contagious and rapidly spreading diseases, so it is necessary to establish awareness sessions and special laboratories to detect the virus and prevent its spread.We note that the estimated values of the survival function decrease as time (t) increases, and this matches the theory about the behavior of the survival function as it is a decreasing function.

10-Recommendations

1. Using new types of weighted distributions due to the flexibility and high efficiency of these distributions in representing time data.
2. Using other estimation methods to estimate the survival function.
3.Application of the new model Nwikpe Distribution Weighted in engineering, medical and industrial aspects11-

References

1. Burnham, K. P., & Anderson, D. R. (2004). Multimodel inference: understanding AIC and BIC in model selection. Sociological methods & research, 33(2), 261-304.
2. Gupta, Isha.(2017) ," Bayesian and E- Bayesian Method of Estimation of Parameter of Rayleigh Distribution- A Bayesian Approach under Linex Loss Function", Department of Statistics, University of Jammu, Volume 12, Number 4, pp. 791-796 , Indi.
3. Hafez, E. H., Riad, F. H., Mubarak, S. A., & Mohamed, M. S. (2020). Study on lindley distribution accelerated life tests: application and numerical simulation. Symmetry, 12(12), 2080.
4. Han, M. E-Bayesian method to estimate failure rate. in the sixth international symposium on operations research and its applications (ISOR06) Xinjiang 299–311 (Citeseer, 2006).

5. Islam, A. F. M. (2011). Loss functions, utility functions and Bayesian sample size determination (Doctoral dissertation).
6. Jasem . Khader Nassif (2012) 'Comparing the estimation of the reliability function of the mixed exponential distribution with a practical application' Ph.D. thesis, College of Administration and Economics - Baghdad.
7. Kamakura, W. A., & Wedel, M. (2004). An Empirical Bayes Procedure for lmproving. JOUrnal Of BusineSS & Economic Statistics, 22(1).
8. Lavanya, A. Bayesian Estimation Of Parameters Survival Function Failure Rate And Auc Under Constant Shape Bi Weibull Distribution Using Failure Time Data2002; 39:449–52.
9. Mayathevar, K., Veluchamy, M., & Subramani, B. (2020). Fuzzy color histogram equalization with weighted distribution for image enhancement. Optik, 216, 164927.
10. . Nwikpe, B. J., & Essi, I. D. (2021). Two-Parameter Nwikpe (TPAN) Distribution with Application. Asian Journal of Probability and Statistics, 56-67.
11. Patil, G. P., & Taillie, C. (1989). Probing encountered data, meta analysis and weighted distribution methods. In Statistical data analysis and inference (pp. 317-345). North-Holland.
12. Le Cam, L. (1990). Maximum likelihood: an introduction. International Statistical Review/Revue Internationale de Statistique, 153-171 .
13. Richards, F. S. (1961). A method of maximum-likelihood estimation. Journal of the Royal Statistical Society: Series B (Methodological), 23(2), 469-475.
14. Tsutakawa, R. K., & Soltys, M. J. (1988). Approximation for Bayesian ability estimation. Journal of Educational Statistics, 13(2), 117-130..
15. Yamaoka, K., Nakagawa, T., & Uno, T. (1978). Application of Akaike's information criterion (AIC) in the evaluation of linear pharmacokinetic equations. Journal of pharmacokinetics and biopharmaceutics, 6(2), 165-175.
16. Yatuma, Harith Salim Zia (2011), 'Estimation of the shape parameter and reliability of the Burr type -XII distribution using type II incremental control data with practical application', Master's thesis, Department of Statistics, College of Administration and Economics, University of Baghdad.