WEIGHTED ZEGHDOUDI DISTRIBUTION: PROPERTIES AND APPLICATIONS

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Abstract

The study seeks to use the theory of weighted distributions to obtain a new proposed probabilistic model known as the This paper proposes a one-parameter weighted ZeghdOudi distribution (WZD) which it is created on mixtures of the gamma (2, θ) and gamma (3, θ) distributions. Its raw moments and principal moments have been obtained. The moment based measures including ,coefficient of variation, :skewness, kurtosis then index of diffusion have been discussed. The statistical properties including; hazard rate function, mean residual life function and stochastic ordering have stood described, Maximum likelihood estimation has stood discussed aimed at estimating the parameters of the distribution, Finally, applications of the distribution have stayed explained with three examples of experiential real lifetime.

Keywords: Weighted Distribution, ZeghdOudi distribution., Moments, Statistical properties. Maximum Likelihood estimation, Goodness of fit.

1. Introduction

"The statistical researcher faces many statistical problems during the process of analyzing the data as well as estimating the parameters related to the distribution, and among those problems that the researcher directs is the process of determining the appropriate and appropriate distribution of the data phenomenon The researchers specialized in the field specialized in the statistical field have worked to develop the probability distributions and move them to the stage of the weighted probability distributions, in order to obtain the best representation of the data with the least errors, especially when the researcher faces the problem of choosing the sample with an equal probability, which made the original distribution limited to the modeling of phenomena It is useless and when it becomes necessary to suggest a specific modification to be done through the weights system in order to obtain vocabulary that has the same appearance of accidents. ZeghdOudi distribution (WZD) is a newly proposed lifetime distribution introduced by Hamada (2018) of which one parameter ZeghdOudi distribution. Hamada has also debated its, several mathematical , statistical properties with its shape, moments, order statistics, Bonferroni and Lorenz curve, Skewness, Kurtosis, mean deviations, Reni entropy, hazard function, stochastic ordering, mean residual life function and Stress strength reliability. Hamada has also discussed its estimation of parameters via using the method of moments and maximum likelihood estimation. Likewise: stood discussed for estimating its parameter. Lastly, the goodness. of fit test, using K-S .Statistics (Kolmogorov-Smirnov statistics) for four real generation data- sets have been presented to demonstrate the applicability and comparability of ZeghdOudi, Gamma. Weibull,, Lognormal, Arad Hana, Akashi, Shankar and Lindley and exponential distributions for modelling lifetime data. The probability density function. of ZeghdOudi distribution is arming combination of gamma (2, θ) and gamma (3, θ) distributions. “

The Probability density function of ZeghdOudi distribution (ZD) is given by:

$$f(x,\theta)=\frac{\theta^3xe^{-\theta x}+\theta^3x^2\ \ e^{-\theta x}}{2+\theta};x>0,\theta>0 \qquad (1)$$

and the cumulative distribution function of the ZeghdOudi distribution (ZD) is given by:

$$F(x,\theta)=1-\left(\frac{\theta^2x^2\ +\theta(2+\theta)x+\theta+2}{2+\theta}\right)e^{-\theta x};x>0,\theta>0 \qquad (2)$$

2-Weighted ZeghdOudi Distribution (WZD)

A newly introduced concept of distributions known as weighted probability distributions was given by Fisher (1934) to classical the ascertainment bias. Later Rao in 1965 advanced this concept in a unified means while modelling the statistical data, when the standard distributions were not appropriate to record these observations with equal probabilities. As a result, weighted models stood formulated in such situations to record the observations according to some weighted function. The weighted distribution reduces to length biased distribution as the weight function considers only the length of the units. The concept of length biased sampling, was first presented by Cox (1969) and Helen (1974). The weighted distributions ascend after the observations generated from a stochastic;;; procedure are not agreed equal chance of being recorded,; instead they are recorded rendering to some weighted function. More generally, when the sampling mechanism picks units with probability proportional to measure of the unit size, resulting distribution is termed size-biased distributions. Size biased distributions are a unusual case of the additional general form identified as weighted distributions, The usefulness and applications of weighted distributions to biased samples in various areas including medicine, ecology.reliability, and forking processes can be seen in Patel and Rao (1978). Gupta and Kirmani (1990). Gupta and Keating (1985). Different authors have reviewed and studied the various weighted probability models and demonstrated their applications in different fields. Weighted distributions were applied in various research areas related to dependability, biomedicine, ecology and branching processes. Shankar (2017) discussed a Size-Biased Poisson-Shanker Distribution and its applications to handle various count data cliques, Recently Shankar and Shukla (2018) discussed a new generalized size-biased, Poisson-Lindley distribution with its applications to faultless size distribution. Also, Subramanian and Rather (2018) obtained the weighted version of exponentiated mukherjee-islam distribution by statistical properties. Rather and Subramanian (2019) discussed on weighted sus hila distribution with properties and applications which shows more suppleness than the subject distribution. Tonsures deb roy et.al (2011) have obtained the length biased weighted Weibull distribution. Recently, Ganaie, Rajagopalan and Rather (2020) discussed weighted new-fangled quasi Lindely distribution with properties and requests."

Assume t is a non-negative random variable with probability density function f (t) ..Let t^c be the non-negative weight function, then, the probability density function of the weighted random variable T w is given by:

$$f_W(c,\theta,x) = \frac{t^c f(x,\theta)}{E(t^c)} \tag{3}$$

Where:

$$E(t^c) = Ex^r = \int_0^\infty x^r \, f(x,\theta)dx$$

$$= \int_0^\infty x^r \frac{\theta^3 x e^{-\theta x} + \theta^3 x^2 \ \ e^{-\theta x}}{2+\theta} dx$$

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$$=\frac{1}{2+\theta}\int_0^\infty \theta^3 x^{r+1}e^{-\theta x}+\theta^3 x^{r+2}\ e^{-\theta x}dx$$

$$=\frac{1}{2+\theta}\left[\theta^3\int_0^\infty x^{r+1}e^{-\theta x}dx+\theta^3\int_0^\infty x^{r+2}\ e^{-\theta x}dx\right]$$

$$=\frac{1}{2+\theta}\left[\theta^3\int_0^\infty \left(\frac{u}{\theta}\right)^{r+1} e^{-u}\frac{du}{\theta}+\theta^3\int_0^\infty \left(\frac{u}{\theta}\right)^{r+2}\ e^{-u}\frac{du}{\theta}\right]$$

$$=\frac{1}{2+\theta}\left[\frac{\theta\Gamma(r+2)+\Gamma(r+3)}{\theta^r}\right]$$

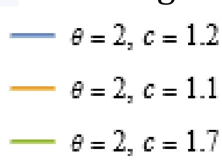
$$=\frac{\theta\Gamma(c+2)+\Gamma(c+3)}{(2+\theta)\theta^c} \qquad (4)$$

Supernumerary equation (1) and (4) in equation (3),, we obtain the probability, density function of weighted Two parameters ZeghdOudi Distribution,.

$$f(x,\theta,c)_w=\left[\frac{\theta^3x^{c+1}e^{-\theta x}+\theta^3x^{c+2}\ \ e^{-\theta x}}{2+\theta}\right]\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}$$

$$f(x,\theta,c)_w=\frac{\theta^{3+c}e^{-\theta x}(x^{c+1}+x^{c+2})}{\theta\Gamma(c+2)+\Gamma(c+3)} \qquad (5)$$

Theorem: of weighted Two parameters ZeghdOudi Distribution is a Proper PDF This implies that:



$$\int_0^\infty f(x,\theta,c)_w dx=\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\int_0^\infty\left[\frac{\theta^3x^{c+1}e^{-\theta x}+\theta^3x^{c+2}\ \ e^{-\theta x}}{2+\theta}\right]dx$$

$$=\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\int_0^\infty\frac{1}{2+\theta}\left[\theta^3x^{c+1}e^{-\theta x}+\theta^3x^{c+2}\ \ e^{-\theta x}\right]dx$$

$$=\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\left(\frac{1}{2+\theta}\left[\theta^3\int_0^\infty x^{c+1}e^{-\theta x}dx+\theta^3\int_0^\infty x^{c+2}\ \ e^{-\theta x}dx\right]\right)$$

$$\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}=k$$

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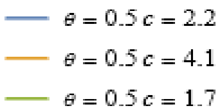
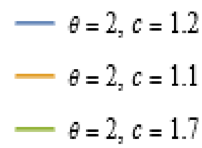
$$= \frac{k}{2+\theta}\left[\theta^{3}\int_{0}^{\infty}\left(\frac{u}{\theta}\right)^{c+1} e^{-u}\frac{du}{\theta} + \theta^{3}\int_{0}^{\infty}\left(\frac{u}{\theta}\right)^{c+2} e^{-\theta x}\frac{du}{\theta}\right]$$

$$= \frac{k}{2+\theta}\left[\theta\int_{0}^{\infty} u^{c+1}e^{-u}du + \int_{0}^{\infty} u^{c+2}\ e^{-u}du\right]$$





$$= \frac{k}{2+\theta}\left[\frac{\theta\Gamma(c+2)+\Gamma(c+3)}{\theta^{c}}\right]$$

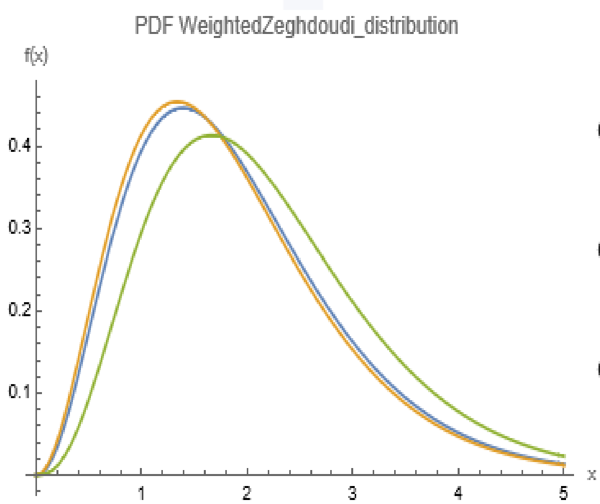
$$= \frac{(2+\theta)\theta^{c}}{\theta\Gamma(c+2)+\Gamma(c+3)}\left[\frac{\theta\Gamma(c+2)+\Gamma(c+3)}{(2+\theta)\theta^{c}}\right]$$

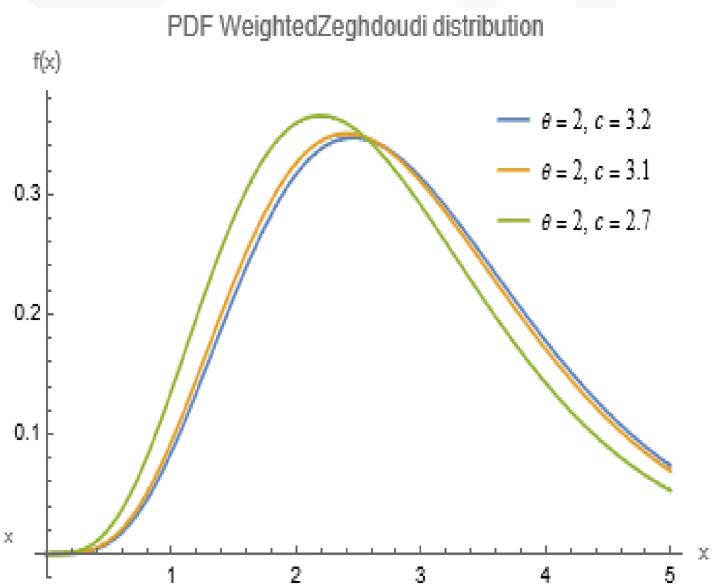
$$= 1$$

The corresponding cumulative distribution function weighted Two parameters ZeghdOudi distribution is assumed by:

$$\mathrm{F}(\mathrm{x},\theta,\mathrm{c})_{\mathrm{w}}\mathrm{dx} = \int_{0}^{\mathrm{x}} \mathrm{f}(\mathrm{x},\theta,\mathrm{c})_{\mathrm{w}}\mathrm{dx}$$

$$\mathrm{F}(\mathrm{x},\theta,\mathrm{c})_{\mathrm{w}}\mathrm{dx} = \frac{2\Gamma(c+2)+c\Gamma(c+2)+\theta\Gamma(c+2)-\Gamma(c+2,\theta x)-\Gamma(c+3,\theta x)}{\theta\Gamma(c+2)+\Gamma(c+3)} \qquad (6)$$





3- RELIABLITY ANALYSIS

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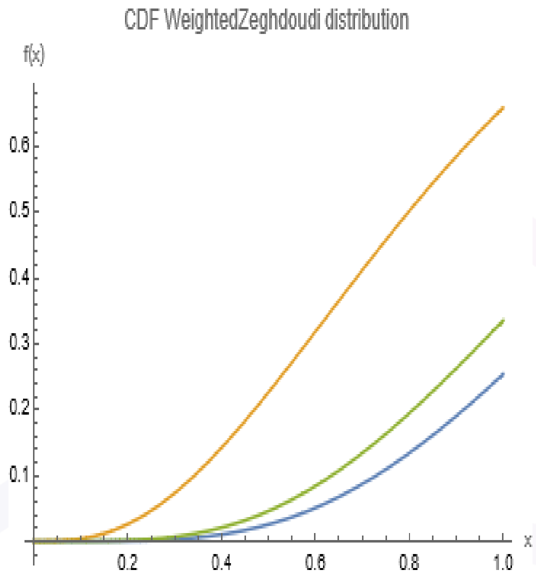
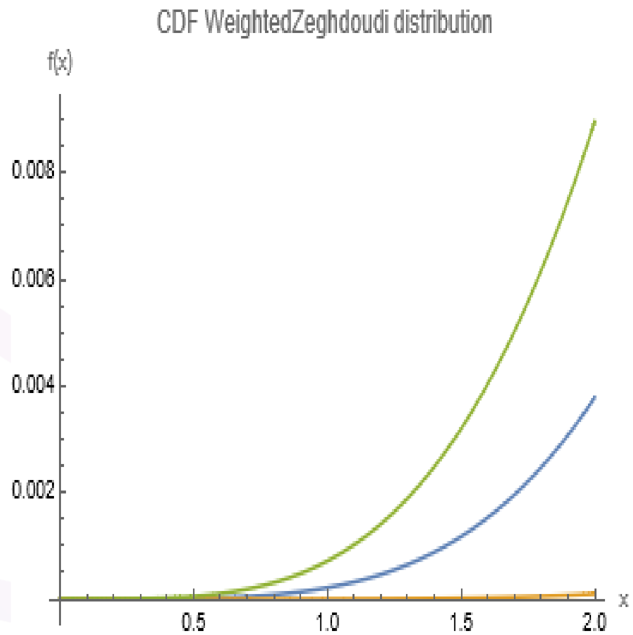


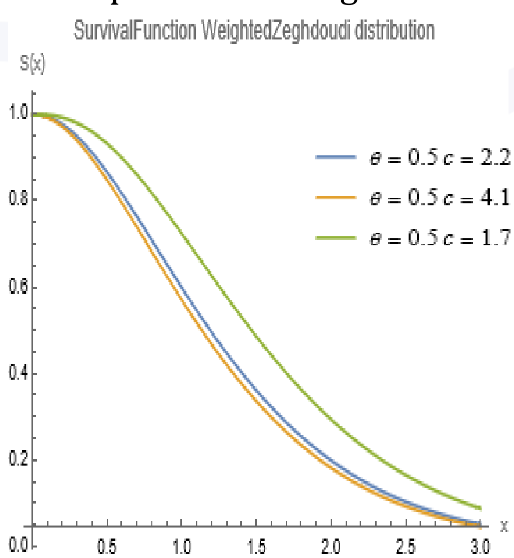


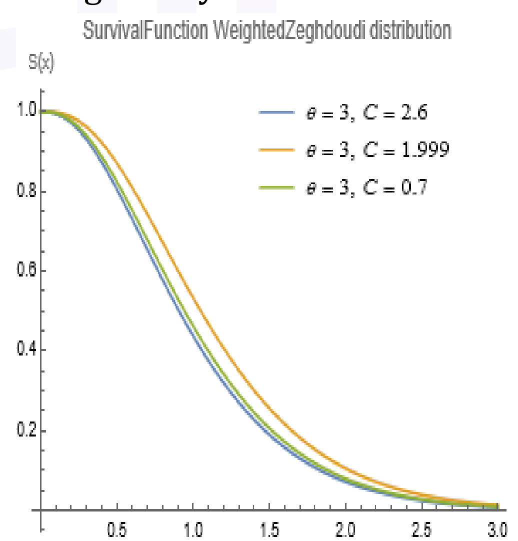
The Survival function of weighted Two parameters ,ZeghdOudi Distribution is calculated as:

$$S(x) = 1 - F(x, \theta, c)_w dx$$

$$S(x) = 1 - \frac{2\Gamma(c+2) + c\Gamma(c+2) + \theta\Gamma(c+2) - \Gamma(c+2, \theta x) - \Gamma(c+3, \theta x)}{\theta\Gamma(c+2) + \Gamma(c+3)} \quad (7)$$

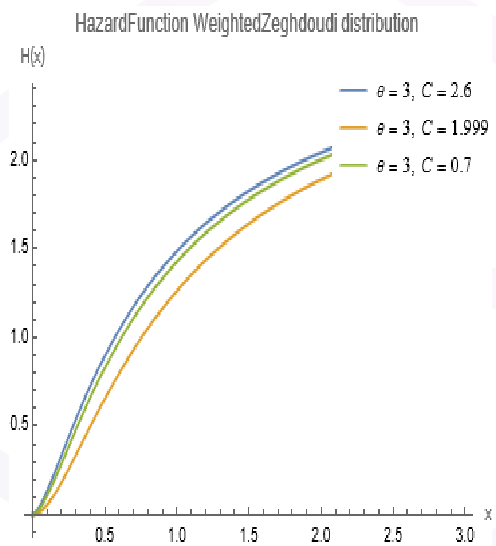
The hazard function is similarly known as hazard rate and is named as the instantaneous failure rate or power of mortality and the hazard function weighted Two parameters ZeghdOudi Distribution is given by

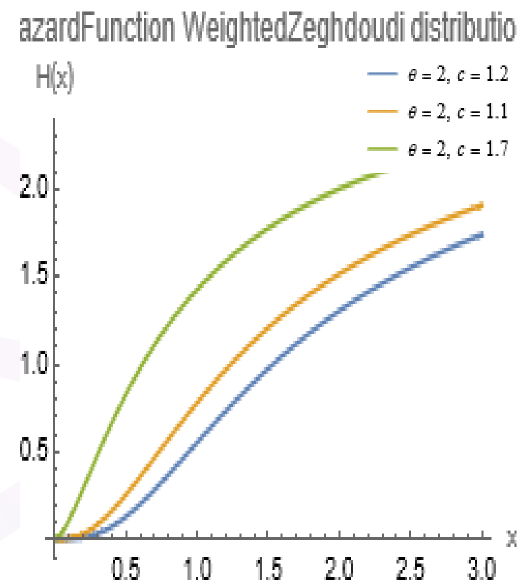




$$H(x) = \frac{f(x,\theta,c)_w}{S(x)}$$

$$H(x) = \frac{\frac{\theta^{3+c}e^{-\theta x}(x^{c+1}+x^{c+2})}{\theta\Gamma(c+2)+\Gamma(c+3)}}{1-\frac{2\Gamma(c+2)+c\Gamma(c+2)+\theta\Gamma(c+2)-\Gamma(c+2,\theta x)-\Gamma(c+3,\theta x)}{\theta\Gamma(c+2)+\Gamma(c+3)}} \qquad (8)$$





4 – STATISTICAL MEASURES

"In this Portion, we have obtained the unlike statistical properties of weighted Two parameters ZeghdOudi Distribution. Moments

Let X denotes the random variable of weighted Two parameters ZeghdOudi Distribution with Parameters θ, and c, then the r th order moment $EX^r$of weighted Two parameters ZeghdOudi Distribution about origin is:"

$$EX^r = \int_0^\infty x^r f(x,\theta,c)_w dx$$

$$= \frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)} \int_0^\infty \frac{1}{2+\theta}\left[\theta^3 x^{c+r+1}e^{-\theta x} + \theta^3 x^{c+r+2}\ e^{-\theta x}\right]dx$$

$$= \frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\left(\frac{1}{2+\theta}\left[\theta^3 x^{c+r+1}e^{-\theta x}dx + \theta^3\int_0^\infty x^{c+r+2}\ e^{-\theta x}dx\right]\right)$$

$$\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)} = k$$

$$= \frac{k}{2+\theta}\left[\theta^3\int_0^\infty \left(\frac{u}{\theta}\right)^{c+r+1} e^{-u}\frac{du}{\theta} + \theta^3\int_0^\infty \left(\frac{u}{\theta}\right)^{c+r+2} e^{-\theta x}\frac{du}{\theta}\right]$$

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$$= \frac{k}{2+\theta}\left[\theta\int_0^\infty u^{c+r+1}e^{-u}du + \int_0^\infty u^{c+r+2}\ e^{-u}du\right]$$

$$= \frac{k}{2+\theta}\left[\frac{\theta\Gamma(c+r+2)+\Gamma(c+r+3)}{\theta^{c+r}}\right]$$

$$= \frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\left(\frac{\theta\Gamma(c+r+2)+\Gamma(c+r+3)}{(2+\theta)\theta^{c+r}}\right)$$

$$Ex^r = \mu'_r$$

$$= \frac{\theta\Gamma(c+r+2)+\Gamma(c+r+3)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^r} \quad (9)$$

:Substitute r = 1,2,3 and 4 in equation (6): we will obtain the first four moments about origin of weighted Two parameters ZeghdOudi Distribution:

$$Ex^1 = \mu'_1 = \frac{\theta\Gamma(c+3)+\Gamma(c+4)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^1}$$

$$Ex^2 = \mu'_2 = \frac{\theta\Gamma(c+4)+\Gamma(c+5)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^2}$$

$$Ex^3 = \mu'_3 = \frac{\theta\Gamma(c+5)+\Gamma(c+6)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^3}$$

$$Ex^4 = \mu'_4 = \frac{\theta\Gamma(c+6)+\Gamma(c+7)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^4}$$

:Using association between dominant moments (moments about the mean), and, moments about origin. the central moments, of weighted Two parameters ZeghdOudi Distribution are obtained as:

$$E(x-\mu)^r = \int_0^\infty (x-\mu)^r f(x,\theta,c)_w$$

$$= \frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\int_0^\infty \frac{1}{2+\theta}(x-\mu)^r\left[\theta^3x^{c+1}e^{-\theta x}+\theta^3x^{c+2}\ e^{-\theta x}\right]dx$$

$$= \frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\left(\frac{(x-\mu)^r}{2+\theta}\left[\theta^3\int_0^\infty x^{c+1}e^{-\theta x}dx+\theta^3\int_0^\infty x^{c+2}\ e^{-\theta x}dx\right]\right)$$

$$\frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)} = k$$

$$= \frac{k}{2+\theta}\sum_{j=0}^{r}\binom{r}{j}x^j(-\mu)^{r-j}\left[\theta^3\int_0^\infty x^{c+1}e^{-\theta x}dx+\theta^3\int_0^\infty x^{c+2}\ e^{-\theta x}dx\right]$$

$$= \frac{k}{2+\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\theta^3\int_0^{\infty} x^{c+1+j}e^{-\theta x}dx + \theta^3\int_0^{\infty} x^{c+2+j}\ \ e^{-\theta x}dx\right]$$

$$= \frac{k}{2+\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\theta^3\int_0^{\infty} x^{c+1+j}e^{-\theta x}dx + \theta^3\int_0^{\infty} x^{c+2+j}\ \ e^{-\theta x}dx\right]$$

$$= \frac{k}{2+\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\theta^3\int_0^{\infty}\left(\frac{u}{\theta}\right)^{c+1+j} e^{-u}\frac{du}{\theta} + \theta^3\int_0^{\infty}\left(\frac{u}{\theta}\right)^{c+2+j} e^{-u}\frac{du}{\theta}\right]$$

$$= \frac{k}{2+\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\frac{\theta\int_0^{\infty}(u)^{c+1+j}e^{-u}du}{\theta^{c+j}} + \frac{\int_0^{\infty}(u)^{c+2+j}e^{-u}du}{\theta^{c+j}}\right]$$

$$= \frac{k}{2+\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\frac{\theta\Gamma c+2+j}{\theta^{c+j}} + \frac{\Gamma c+3+j}{\theta^{c+j}}\right]$$

$$= \frac{(2+\theta)\theta^c}{\theta\Gamma(c+2)+\Gamma(c+3)}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\frac{\theta\Gamma c+2+j+\Gamma c+3+j}{(2+\theta)\theta^{c+j}}\right]$$

$$= \frac{1}{\theta\Gamma(c+2)+\Gamma(c+3)}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\frac{\theta\Gamma c+2+j+\Gamma c+3+j}{\theta^{j}}\right]$$

$$(x-\mu)^r = \frac{1}{\theta\Gamma(c+2)+\Gamma(c+3)}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\frac{\theta\Gamma c+2+j+\Gamma c+3+j}{\theta^{j}}\right] \tag{10}$$

Supernumerary r = 1,2,3 and 4 in equation (7) we will obtain the primary four moments about origin of weighted Two parameters ZeghdOudi Distribution

$$(x-\mu)^2 = \frac{1}{\theta\Gamma(c+2)+\Gamma(c+3)}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\left[\frac{\theta\Gamma c+2+j+\Gamma c+3+j}{\theta^{j}}\right]$$

$$(x-\mu)^2 = \sigma^2 = \frac{\theta\Gamma(c+4)+\Gamma(c+5)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^2} - \left(\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^1}\right)^2$$

$$\sigma = \sqrt{\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^2} - \left(\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{(\theta\Gamma(c+2)+\Gamma(c+3))\theta^1}\right)^2}$$

$$(x-\mu)^3 = \frac{\theta\Gamma(c+5)+\Gamma(c+6)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^3} - 3\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1}\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2} + 2\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2}$$

$$(x-\mu)^4 = \frac{\theta\Gamma(c+6)+\Gamma(c+7)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^4} - 4\frac{\theta\Gamma(c+5)+\Gamma(c+6)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^3}\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1} + 6\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2}\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2} - 3\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1}$$

$$\mathrm{C.V} = \frac{\sigma}{\mu_1}\times 100\%$$

$$\mathrm{C.V} = \frac{\sqrt{\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2} - \left(\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1}\right)^2}}{\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1}}\times 100\%$$

$$\mathrm{S.K} = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}$$

$$\mathrm{S.K} = \left[\frac{\left(\frac{\theta\Gamma(c+5)+\Gamma(c+6)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^3} - 3\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1}\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2} + 2\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2}\right)}{\frac{\theta\Gamma(c+4)+\Gamma(c+5)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^2} - \left(\frac{\theta\Gamma(c+3)+\Gamma(c+4)}{\big(\theta\Gamma(c+2)+\Gamma(c+3)\big)\theta^1}\right)^{\frac{3}{2}}}\right]$$

5-PARAMETER ESTIMATION

The method of maximum likelihood estimate is used for estimating the parameters of the newly proposed distribution known as the weighted Two parameters ZeghdOudi Distribution. Let x1, x2,…,xn be a random sample of size n from the weighted Two parameters ZeghdOudi Distribution, then the corresponding likelihood function is given by

$$Lf(x_1.x_2.\dots.x_n) = \prod_{i=1}^{n} f(x.\theta.\alpha.c) \qquad \dots (11)$$

$$= \left[\frac{\theta^{c+3}e^{-\theta x}(x^{c+1}+x^{c+2}\ )}{\theta\Gamma(c+2)+\Gamma(c+3)}\right]$$

$$= \prod_{i=1}^{n}\left[\frac{\theta^{c+3}e^{-\theta x}(x^{c+1}+x^{c+2}\ )}{\theta\Gamma(c+2)+\Gamma(c+3)}\right]$$

$$= \frac{\theta^{n(c+3)}e^{-\theta\sum_{i=1}^{n}xi}}{\left(\theta\Gamma(c+2)+\Gamma(c+3)\right)^{n}}\prod_{i=1}^{n}[(x^{c+1}+x^{c+2}\ )]$$

$$= \theta^{n(c+3)}e^{-\theta\sum_{i=1}^{n}xi}\left(\theta\Gamma(c+2)+\Gamma(c+3)\right)^{-n}\prod_{i=1}^{n}[(x^{c+1}+x^{c+2}\ )]$$

$$\log Lf(x_1.x_n) = cn\ ln\theta + 3n(ln\theta) - \theta\sum_{i=1}^{n}xi - nlog\left(\theta\Gamma(c+2)+\Gamma(c+3)\right) + \sum_{i=1}^{n}ln(x^{c+1}+x^{c+2})$$

$$\frac{\log Lf(x_1\dots x_n)}{d\theta} = \frac{cn}{\theta}+\frac{3n}{\theta}-\sum_{i=1}^{n}xi-\frac{n\Gamma(c+2)}{\left(\theta\Gamma(c+2)+\Gamma(c+3)\right)}$$

$$\frac{\log Lf(x_1\dots x_n)}{dc} = n\ ln\theta - \frac{n\left(\theta\acute{\Gamma}(c+2)+\acute{\Gamma}(c+3)\right)}{\left(\theta\Gamma(c+2)+\Gamma(c+3)\right)} + \sum_{i=1}^{n}\left(\frac{(c+1)x^{c}}{x^{c+1}}+\frac{(c+2)x^{c+1}}{x^{c+2}}\right)$$

$$\Psi(Z) = \frac{d\ \log(Z)}{dZ} = \frac{\acute{\Gamma}(Z)}{\Gamma(Z)} \quad ; \ Z \neq -1,-2,\dots$$

Equations (2-27), (2-28) and (2-29) cannot be solved by the usual analytical methods because they are non-linear equations and therefore they were solved using the numerical method (Nelder-Mead) to obtain the estimations of the greatest possibility method

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6-Data Analysis

In this section one dataset from Persons with(covid-19) has been considered for testing the goodness of fit of Two parameters ZeghdOudi Distribution. The following dataset has been considered.

TABLE I

0.8	1.6	2.2	2.5	2.8	3.1	3.4	3.8	4.2	5
0.8	1.6	2.2	2.5	2.8	3.1	3.5	3.8	4.3	5.2
0.9	1.7	2.3	2.5	2.8	3.2	3.5	3.9	4.4	5.3
1.1	1.7	2.3	2.5	2.9	3.3	3.5	4	4.5	5.5
1.1	1.8	2.3	2.6	2.9	3.3	3.6	4	4.5	6
1.3	1.8	2.3	2.6	3	3.3	3.6	4	4.6	6.2
1.4	1.8	2.3	2.6	3	3.4	3.6	4	4.7	6.3
1.4	1.9	2.4	2.6	3	3.4	3.6	4	4.8	7
1.5	2	2.4	2.6	3	3.4	3.6	4	4.8	7.2
1.5	2	2.4	2.7	3.1	3.4	3.6	4.2	4.9	8

The people of people with Fisheros (covid-19 are represented in Karbala, measured clocks size (n=100) , measured clocks

“For these three datasets weighted Two parameters ZeghdOudi Distribution has remained fitted along by one parameter ;Lindley distribution (LD) .and ExponentialDistribution,, two-parameter Gamma distribution and ZeghdOudi Distribution” , The ML estimates, values of Anderson- Darling,” Cramer-von Mises, Pearson$\chi^2$and p-value of the fitted distributions are presented in tables 3.

TABLE III

Distributions	MLE	X^2Anderson-D		Cramer- V		Pearson	
		statistic	P-Value	statistic	P-Value	statistic	P-Value
weighted Z.D	$\hat{\theta} = 0.0324$ $\hat{c} = 1.62987$	0.7209	0.54067	0.096413	0.60274	10.59	0.39032
Z.D	$\hat{\theta} = 0.021$	1.9654	0.0961	0.32604	0.1143	24.655	0.006
Lindley.D	$\hat{\theta} = 0.9279$	13.62	0.325629	0.41616	0.0655096	2.45691	0.0523163
EX.D	$\hat{\lambda} = 0.3093$	6.1009	0.072877	1.0715	0.0016741	39.8	0.0000752
Gamma	$\hat{\alpha} = 5.1656$ $\hat{\theta} = 0.6258$	0.2638	0.26252	0.04329	0.91549	8.42	0.1515

ML Estimates and Criterion Values X^2Anderson-D, Cramer- V, and Pearson and comparison of weighted Two parameters ZeghdOudi Distribution with ZeghdOudi Distribution, Exponential, Gamma distribution Two parameters and Lindley One parameters Distribution.

6- CONCLUSION

" In the present manuscript, we have deliberate a new version of one parameters ZeghdOudi distribution known as weighted two parameters ZeghdOudi distribution. The subject distribution is generated by using the weighting technique and taking the one parameter ZeghdOudi distribution as the base distribution. About mathematical and Statistical properties, of the afresh proposed distribution are derived and discussed. The application of the newly introduced distribution has also stood demonstrated by real life data set and the outcome .of the data sets presented that the weighted Two parameters ZeghdOudi Distribution fits better over one parameters ZeghdOudi. Distribution".

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