ISSN: 2776-0960 Volume 3, Issue 9 Sep., 2022

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

M – OPEN NANO TOPOLOGICAL SPACES

Nawras Hasan Mohammed
Corresponding author Department of Mathematics College of Education for Pure Sciences,Tikrit University
E-mail: nawras.h.mohammed@st.tu.edu.iq

Ali A. Shihab
Corresponding author Department of Mathematics
College of Education for Pure Sciences,Tikrit University

Prof. Dr. Ali Abdul Al Majeed Shihab,
Lecturer Mathematics College of Education for Pure Science,
Mathematics Department, University of Tikrit, P.O.Box, 42,Tikrit.Iraq
draliabd@tu.edu.iq, ali.abd82@yahoo.com

Abstract
The objective of this paper is to present a new definition of Nano-open sets called Nano_M_open sets(M_N-OS) and study their components and features with some examples. To reach relations with many types of M_N-OS are studied. He also proved that the family of M_N-OSs forms a topology on universe set U is called Nano-M topological space(M_N_TS) on U.

Keywords: Nano-open sets, Nano $-M$ $-$open sets

Introduction
In 1963 Levine [1] establish the notion of semi-open sets. In 1985 Njastad [2] establish the notion of alpha-open sets, pre-open sets [3], δ-open set [4], θ-semi open set [5], Regular –open set [6], θ-open set [7].In 1983 Abd ElMonsef et al. [8] introduce the notion of β-open set. In 2013 Thivagar M. Lellis [9] introduce idea of Nano-topological space(N_TS) with respect to a subset X of universe U which is defined as an upper and lower approximation of X.Element of N_TS are called a Nano-open sets (N_OS).El-Maghrabi, A.I. and AL.Jahani Mohammad in 2011 [10] establish the notion of M $-$open set and we will know a new definition of Nano-topological space.

Preliminaries

A subset A of a space (X,τ) is called semi-open(Se_O.) [1] (resp. α-open(α_O.) [2], β-open(β_O.) [8], preopen (Pr_O.) [3], δ-open(δ_O.) [4], θ-open(θ_O.) [7], Regular-open(Re_O.) [6], θ- semi –open(θ_S_O.) [5]) set if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$resp.[$A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$ $A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$, $A \subseteq \mathrm{int}(\mathrm{cl}(A))$, $A = \mathrm{int}_\delta(A)$, $A = int_\theta(A)$,$A = \mathrm{int}(\mathrm{cl}(A))$,$A \subseteq \mathrm{cl}(int_\theta(A))$.The complement of Se_O (resp. α_., β_, Pr_, δ_, θ_, Re_, θ_S_) O. set is said to be semi- closed(Se_C.)(resp. α-,β-, pre,δ_,θ_,Regular-,θ_S_) closed set. Intersection of Se_C. (resp. α-, β- , Pr_, δ-, θ-, Re- , θ_S_)C. sets continuing $A$is the semi- (resp. α- , β-, pre, δ- closed, θ-, Regular-, θ- semi-) closure, is dented by$Scl(A)$[resp. $\alpha cl(A)$, $\beta cl(A)$, $Pcl(A)$,$\theta cl(A)$, $Rcl(A)$, $\delta cl_\theta(A)$]. The union of all Se_O. (resp. α_O., β_O., Pr_O., δ_O., θ_O., Re_O., θ_S_O.) sets contained in A is said semi- (resp. α-, β-, pre, δ-, θ-, Regular-, θ- semi -) interior, is briefly by $Sint(A)$ [αint, βint,$Pint$,θint,$Rint$, δint_θ] (A) The sets of all Se_O. (resp. α_O., β_O., Pr_O., δ_O., θ_O., Re_O., θ_S_O.) set is briefly $SO(X)$[resp. αO, βO, PO,θO, RO , δO] (X).

Definition 2.1 [9]: U is a non-empty finite set of elements, called the universe and R be an equivalence relation on U named as the indiscernibility relation. The elements in the same class, are said to be indiscernible with one another. The binary (U,R) is called the approximation space. Let $X \subseteq U$

a) $L_{R(X)} = \cup_{X\in U} \{R(X): R(X) \subseteq X\}$. is the lower approximation of with respect to equivalence class $R(X)$.

b) $U_R(X) = \cup_{X\in U} \{R(X): R(X) \cap X \neq \emptyset\}$is the upper approximation of X with respect to $R(X)$.

c) $B_{R(X)} = U_{R(X)} - L_{R(X)}$is boundary region of X with respect to $R(X)$.

Definition 2.2 [9]: let U is the universe, R be an equivalence relation on U. $T_R(X) = \{U, \phi, L_{R(X)}, U_{R(X)}, B_{R(X)}\}$, $X \subseteq U$. $T_R(X)$ check axioms:

1- U & $\emptyset \in T_R(X)$.

2- The union of objects of any sub sets of $T_R(X)$ is $T_R(X)$.

3- The intersection of the objects of any sub collection of $\mathrm{T_R(X)}$ is $\mathrm{T_R(X)}$. Thus $\mathrm{T_R(X)}$ is topology on U, be called N_T. on U with respect to X. we call $(\mathrm{U}, \mathrm{T_{R(X)}})$ as the N_TS. The members of $\mathrm{T_R(X)}$ be called a N_O.

We note that, if $\mathrm{T_R(X)}$ is N_T. on U, we get $\beta = \{\mathrm{U}, \mathrm{L_{R(X)}}, \mathrm{B_{R(X)}}\}$ basis for $\mathrm{T_R(X)}$.

Definition 2.3 [9]: let $(U, T_R(X))$ is a N_TS. with respect to X, where $X \subseteq U$, $A \subseteq U$:1-Interior. Then the Nano-interior of A defined as the union of all N_O. subset of A and symbolized by Nint(A) that is Nint(A) is the largest N_O. subset of A. 2-Closure. The Nano-closure of A is the intersection of all N_C. sets containing A and denoted by Ncl(A) that is Ncl(A) is the smallest N_C. set containing A.

Definition 2.4 [9]: let $(U, T_R(X))$ is N_TS. , $A \subseteq U$, the A is

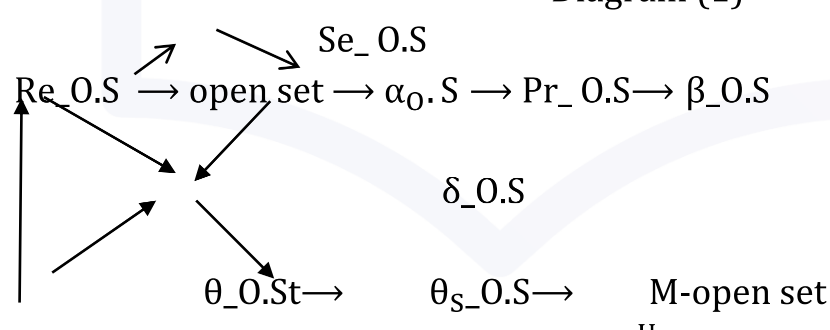
1- Nano-Semi-open(NSe_O.)if $A \subseteq Ncl(Nint(A))$.
2- Nano-pre-open(NPr_O.)if$A \subseteq int(Ncl(A))$.
3- Nano-δ-open (Nδ_O.)if $A \subseteq \overline{A^{\circ}_{\delta}}$.[4]
4- Nano-θ-semiopen($N\theta_S$_O.) if $A \subseteq \overline{A^{\circ}_{\theta}}$.[4]

Definition 2.5 [9]: let $(U, T_R(X))$ a N_TS., $A \subseteq U$, then A be called NSe_C. (NPr_C., Nα_C., and NRe_C.) if its complement is NSe_O. (NPr_O.open, Nα_O., and NRe_O.respectively).

Definition 2.6 [8]: let $A \subseteq (U, T_R(X))$ is Nβ_O. on U if $A \subseteq Ncl(Nint(Ncl(A)))$. The set of all Nβ_O. sets of U denoted by NβO(U, X).

Definition 2.7 [11]: $T_R(X)$ is N_T. on U with respect to X.$A \subseteq U$ is Nano-θ-open denoted by (Nθ_O.) if for each $x \in A, \exists$ G is N_OS. $\ni x \in G \subseteq Ncl(G) \subseteq A$.

Diagram (1)



Example 2.8: Let $U = \{1,2,3,4\}$ with $\frac{U}{R} = \{\{1\},\{2,3\},\{4\}\}$ and $X = \{1,2\}$. Then $T_R(X) = \{U, \emptyset, \{1\}, \{1,2,3\}, \{2,3\}\}$, the N_C. sets are, $N_C(U, X) = \{U, \phi, \{4\}, \{1,4\}, \{2,3,4\}\}$ then $N_{\delta o}(U, X) = \{U, \emptyset, \{1\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{2,3,4\}\}$,$N_{po}(U, x) = \{U, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$,

$\{1,2,3\},\{1,3,4\},\{1,2,4\}\}$, $T_R^{\alpha}(X) = \{U, \emptyset, \{1\}, \{2,3\}, \{1,2,3\}\}$ and $N_{Ro} = \{U, \emptyset, \{1\}, \{2,3\}\}$ then $T_R^{\alpha}(X)$ form topology on U, but $N_{\delta o}(U, X)$ does not form topology on U, since {1,4} and {2,3,4} are NSe_O. sets but $\{1,4\} \cap \{2,3,4\} = \{4\} \notin N_{\delta o}(U, X)$. Also $N_{po}(U, X)$ does not form topology on U. Since {1,3,4} and {1,2,4} are NPr_. O.but $\{1,3,4\} \cap [1,2,4] = \{1,4\} \notin N_{po}(U, X)$ and $N_{Ro}(U, X)$ does not form topology on U. Since {1} and {2,3} are NRe_O.sets but $\{1\} \cup [2,3]$ are NRe_O. but $\{1\} \cup [2,3] = \{1,2,3\} \notin N_{Ro}(U, X)$.

Definition 2.9: let $(U, T_R(X))$ is a N_TS. Then $A \subseteq U$ is Nano-M-open set in a N_TS. if $A \subseteq Cl_N(N\,int_\theta(A) \cup int_N(N\,Cl_\delta(A))$ briefly M_N-O.

Definition 2.10 [12]: let $N\,int_\theta = \cup \{B \in T_N : \overline{B_N} \subseteq A$ such that $x \in B \in T_N\}$.

Definition 2.11 [4]: let $N\,Cl_\delta = \cup \left\{x \in U : \overline{B_N}^{\circ} \cap A \neq \emptyset \text{ such that } B \in T_N, x \in B\right\}$

Remark 2.12 [10]: The opposite is not necessarily true as shown in the following examples.

Example 2.13: let $X = \{1,2,3,4\}$ with $T = \{X, \emptyset, \{1\}, \{3\}, \{1,3\}\}$. We get {1} is M-O. ,but not θ-Semi open.

Example 2.14: let $X = \{\mathcal{P}, \hbar, g\}$ and $T = \{X, \emptyset, \{\mathcal{P}\}, \{\hbar\}, \{\mathcal{P}, \hbar\}\}$.Then $\{\hbar, g\}$ is an M-OS, but not δ-pre open.

Remark 2.15: The intersection of any two M-O. sets is not M-O. So $X = \{\mathcal{P}, \hbar, g\}$, $T = \{X, \emptyset, \{\hbar\}, \{g\}\{\hbar, g\}\}$. Then $A = \{\mathcal{P}, g\}$ and $B = \{\mathcal{P}, \hbar\}$ are M-O. sets, but $A \cap B = \{\mathcal{P}\}$ is not M-O.

Example 2.16: Let $U = \{1,2,3,4\}$, $T_R(X) = \{U, \emptyset, \{1,2,3\}\}$, $T_R^C(X) = \{\emptyset, U, \{4\}\}$.

A	$\overline{A_N}$	$\mathring{A}_N$	$Nint_\theta(N_{o_\theta})$	$\overline{Nint_\theta}\ \overline{(N_{o_\theta})}$	$Cl_\delta(A)$	$Nint(NCl_\delta)$
1	U	∅	∅	∅	1	∅
2	U	∅	∅	∅	2	∅
3	U	∅	∅	∅	3	∅
4	4	∅	∅	∅	4	∅
12	U	∅	∅	∅	12	∅
13	U	∅	∅	∅	13	∅
14	U	∅	∅	∅	14	∅
23	U	∅	∅	∅	23	∅
24	*U*	∅	∅	∅	24	∅
34	U	∅	∅	∅	34	∅
123	U	123	∅	∅	123	123
124	U	∅	∅	∅	124	∅
234	U	∅	∅	∅	234	∅
134	U	∅	∅	∅	134	∅
U	U	U	U	U	U	U
∅	∅	∅	∅	∅	∅	∅

$$\{\mathbf{1}\} \subseteq \emptyset \cup \emptyset = \emptyset$$

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

$\{1,2,3\} \subseteq \emptyset \cup \{1,2,3\} = \{1,2,3\}$ $U \subseteq U \cup U$,then$\{\emptyset, U, \{1,2,3\}\} = M - N_o$

Example 2.17: Let $U = \{a, b, c, d, e\}$

$$U/R = \{\{b\}, \{a, b, e\}, \{a, e\}\}, X = \{b, e\}$$

$T = \{U, \emptyset, \{b\}, \{a, b, e\}, \{a, e\}\}$,$L_R = \{b\}$

$$U_R = U\{R_{(X)} \cap X \neq \emptyset\} = \{a, b, e\}$$

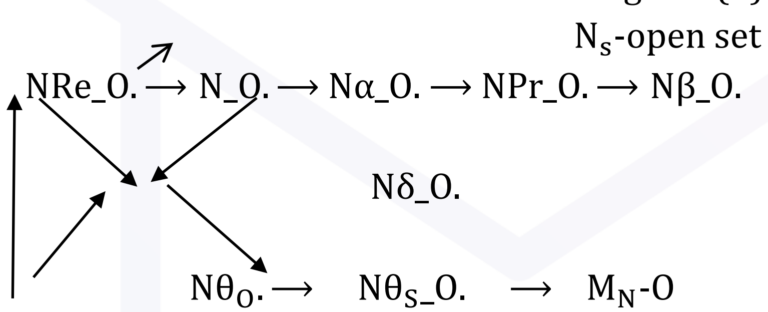
$B = U_R - L_R = \{a, e\}$.In the same way in the above example ,the following solution can be reached $T_{R(X)} = \{\emptyset, U, \{b\}, \{a, b, e\}, \{a, e\}\}$

$T^C_{R(X)}$ =$\{ U, \emptyset$,{a,c,d,e},{c,d},{b,c,d}}. Then $M - N_o$={$\emptyset$,{, U, {a}, {b}, {a, e}, {a, b, e} ,{a,c,d,e},{b,c,d}}

Remark 2.18: Every N-OS is M_N-O.

Result: Nano open $\rightleftarrows$ M_N-O. set.

Diagram (2)



Proposition 2.19: A subset A of $(U, T_R(X))$ then:

1- Every $N\theta_S$_O. is M_N-O. set.

2- Every δ_{Np}-open set is M_N-O.

Proof: (1) suppose that A is Nθ_O. set by (Remark 2.10) A is δ_N-open set and A is $N\theta_S$_O. set. Hence, $A \subseteq int_N\left(Cl_{N_\delta}(A)\right)$ and $A \subseteq Cl_{N_\delta}\left(int_N semi(A)\right)$ then $A \subseteq \overline{\mathring{A}_{\theta_\delta}} \cup \overline{\mathring{A}_\delta}$. By definition 2.1 we get A is M_N-O. set. (2) suppose that A is Nθ_O., since we know Nθ_O. are Nδ_O.& $N\theta_S$_O. by (Remark 2.25). Nθ_Oset is $N\theta_S$_O. & Nδ_O. is δ_{Np}-open set by definition Nθ_O. and δ_{Np} we get $A \subseteq \overline{\mathring{A}_\theta}$ and $A \subseteq \overline{\mathring{A}_\delta}$ then $A \subseteq \overline{\mathring{A}_\theta} \cup \overline{A_\delta}$ and A is M_N-O. set.

Proposition 2.20: If A is an M_N-O.of a $(U, T_R(X))$ & We get A is δ_{Np}-open

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

Proof: Let A be M_N-O. , since $\mathring{A}_\theta = \emptyset \Longrightarrow \overline{\mathring{A}_\theta} = \overline{\emptyset} = \emptyset$. Hence, $A \subseteq \overline{\mathring{A}_\delta} \cup \emptyset = \overline{\mathring{A}_\delta}$ we get $A \subseteq \overline{\mathring{A}_\delta}$. Then A is δ_{Np}-open.

Lemma 2.21: Let $(U, T_R(X))$ be a N_TS. Then:1- union of arbitrary M_N-OS.s is M_N-O. 2- intersection of arbitrary M_N-CS.s is M_N-C.

Proof:

1- $\{A_i, i \in I\}$ is collection of M_N-O.set. We get $A_i \subseteq Cl_N(Nint_\theta(A_i)) \cup int_N(NCl_\delta(A_i))$, such that

$$\cup_i A_i \subseteq \cup_i (Cl_N(N\, int_\theta(A_i))) \cup int_N(NCl_\delta(A_i))$$
$$\subset Cl_N N\, int_\theta(\cup_i A_i) \cup int_N(NCl_\delta(\cup_i A_i))$$

$\forall\, i \in I \rightarrow \cup_i A_i$ is M_N-O.

2- $\{A_i, i \in I\}$ collection of M_N-C. . So $A_i \subseteq Cl_N(Nint_\theta(A_i)) \cap int_N(NCl_\delta(A_i))$, when $\cap_i A_i \subseteq \cap_i (Cl_N(N\, int_\theta(A_i))) \cap int_N(NCl_\delta(A_i)) \subset Cl_N N\, int_\theta(\cap_i A_i) \cap int_N(NCl_\delta(\cap_i A_i)), \forall\, i \in I$, Thus $\cap_i A_i$ is M_N-closed.

Remark 2.22: intersection of any two M_N-OS.s is not M_N-O.

From above example if $U = \{1,2,3\}, T_R(X) = \{x, \emptyset, \{2\}, \{3\}, \{2,3\}\}$. Then $A = \{1,3\}$ and $B = \{1,2\}$ are M_N-open sets. But $A \cap B = \{1\}$ not M_N-open.

Definition 2.23: let $A \subseteq X$. The union of δ-pre open sets contained in A is called the δ-pre-interior $(Pint_\delta(A))$.

Theorem 2.24: Let $(U, T_R(X))$ be N-TS. & $A \subset U$. Then the following data are equivalent:

(1) A is an M_N-O. set.

(2) $A = N_\delta int_\theta(A) \cup NPint_\delta(A)$

Proof: $(1) \rightarrow (2)$. A is M_N-OS. We get $A \subseteq Cl_N(N\, int_\theta(A)) \cup int_N(N\, Cl_\delta(A))$. Hence by proposition 2.26 and lemma 2.27

$$\delta int_\theta(A) \cup Pint_\delta(A) = (A \cap Cl_N(N\, int_\theta(A)) \cup (A \cap int_N(N\, Cl_\delta(A))$$
$$= A \cap Cl_N(N\, int_\theta(A)) \cup int_N(N\, Cl_\delta(A)) = A$$

$(2) \rightarrow (1)$. Suppose that $A = \delta int_\theta(A) \cup Pint_\delta(A)$, then by proposition 2.19 and lemma 2.21

$$A = (A \cap Cl_N(N\, int_\theta(A)) \cap int_N(N\, Cl_\delta(A)) \subset Cl_N(N\, int_\theta(A)) \cup int_N(N\, Cl_\delta(A))$$

Therefore, A is M_N-O.

Proposition 2.25: Let $(U, T_R(x))$ be N-TS. & $A \subset X$. Then the data is equal:

1- A is an M_N-CS.

2- $A = N\delta\, Cl_\theta(A) \cap Np\, Cl_\delta(A)$.

Proof: 1 → 2. A is M_N-OS. Get $A \subseteq Cl_N(N\, int_\theta(A)) \cap int_N(N\, Cl_\delta(A))$. Hence, by proposition 2.26 and lemma 2.27

$$N\delta\, int_\theta(A) \cap Pint_\delta(A) = (A \cap Cl_N(N\, int_\theta(A)) \cap (A \cap int_N(N\, Cl_\delta(A))$$
$$= A \cap Cl_N(N\, int_\theta(A)) \cap int_N(N\, Cl_\delta(A)) = A$$

(2) → (1). Suppose that $A = N\delta\, int_\theta(A) \cap Pint_\delta(A)$, then by proposition 2.19 and lemma 2.21

$A = (A \cap Cl_N(N\, int_\theta(A)) \cap (A \cap int_N(N\, Cl_\delta(A)) \subset Cl_N(N\, int_\theta(A)) \cap int_N(N\, Cl_\delta(A))$.Therefore, A is M_N-O.

Lemma 2.26: $A \subseteq (U, T_R(x))$, where

(1) $M_N - Cl(A) = N\delta\, Cl_\theta(A) \cap Np\, Cl_\delta(A)$

(2) $M_N - int(A) = N\delta\, int_\theta(A) \cup p\, int_\delta(A)$.

Theorem 2.27: Let $A \subset (U, T_R(x))$. we get 1-A is an M_N-OS. iff $A = M_N - int(A)$.
2-A is an M_N-CS. iff $A = M_N - Cl(A)$.

Proof: 1- A is an M_N-OS. Get $A = N\delta\, int_\theta(A) \cup Np\, int_\delta(A)$ by using lemma 2.26, get $A = M_N - int(A)$

Conversely,$A = M_N - int(A)$, using lemma 2.21, $A = N\delta\, int_\theta(A) \cup Np\, int_\delta(A)$, by theorem 2.24, A is M_N-OS.

2-A is an M_N-CS, by theorem 2.24, $A = N\delta\, int_\theta(A) \cap Np\, int_\delta(A)$ & lemma 2.21 we get $A = M_N - int(A)$. Conversely, since $A = M_N - int(A)$.by lemma 2.21, $A = N\delta\, int_\theta(A) \cap Np\, int_\delta(A)$ & by theorem 2.24, A is M_N-CS.

References

1. N. Levine, ''Semi-open sets and semi-continuity in topological spaces'', Amer. Math. Monthly 70, pp.36-41, 1963.
2. O. Njastad, ''On some classes of nearly open sets'', Pacific J. Math. 15, pp. 961-970, 1965.
3. M. H. Stone, ''Applications of the theory of Boolean rings to general topology'', Trans.Amer. Math. Soc. 41(3), pp.375-481, 1937.

4. V. Pankajam and K.Kavitha,''δ-open sets and δ- nano continuity in δ nano topological space'', International Journal of Innovative Science and Research Technology, Vol.2,12, pp.110-118, 2017.
5. M. Caldas, M. Ganster, D. N. Georgiou, S. Jafari and T. Noiri,''θ-semiopen sets and separation axioms in topological spaces'', (submitted).
6. N. V. Velicko, ''H-closed topological spaces'', Amer. Math. Soc. Trans. 78, pp.103-118, 1968.
7. M. Caldas, S. Jafari and M. M. Kovár, ''Some properties of θ-open sets'', Divulg. Mat. 12(2), pp.161-169, 2004.
8. A. Refathy and I. Gnanambal, ''On Nano β-open sets'', Int. Jr. of engineering, contemporary Mathematics and Sciences. Volume 1, No 2, 2015.
9. M. L. Thivagar, C. Rechard. ''On Nano Forms of Weekly Open Sets''. International Journal of Mathematics and Statistics Invention. Volume 1, No 1, pp.31-37, 2013.
10. A.I. El-Maghrabi and M. AL-Juhani, ''M-open sets in topological spaces'', Pioneer Journal of Mathematics and Mathematical Sciences Vol.4, No,2, pp.213-230, 2011.
11. C. Richard, ''Studies on Nano topological spaces'', Ph.D. Thesis, Madurai Kamaraj University, India, 2013.
12. A. S. Mashhour, M. E. Abd EL-Monsef and S. N. EL-Deeb, ''on pre-topological space'', Bull. Math de la Soc. R.S. de Romaine, Volume 28, No 76, PP 39-45, 1984.