ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org
ISSN: 2776-0960 Volume 3, Issue 10 Oct., 2022

SOLVE PARTIAL DIFFERENTIAL EQUATIONS USING PREDICTORS OF EQUATION PARAMETERS

Taqi Muhammad Khattab Abdullah Al-Rubaie
[1]Bachelor's degree from Al-Furat Al-Awsat Technical University, Najaf Branch
Master's degree from Imam Reza University, Mashhad
kaboos287@gmail.com

Mohammad Ghorbanzadeh
Department of Mathematics, Imam Reza International University,
Mashhad, Iran
ghorbanzadeh_imamreza @yahoo.com

Abstract

Partial differential equations model phenomena in physics, engineering, biology, chemistry, physics. nances and other sciences. The parameters of these problems are, in many cases, the coefficients that appear in the Partial Differential Equations that model these problems. For linear equations homogeneous, it is theoretically possible to find these coefficients from a group of solutions of these EDPs.

In a practical situation in which we want to know the coefficients of an EDP to from observations of the phenomenon it models, which characterizes an inverse problem, the solutions tions observed would not be exact, they would be an approximation, possibly with measurement errors, or noise, of exact solutions. In this way we will only get an estimate of the coefficients. Thus, we need to know the behavior of the estimators used when applied to these approximations and in relation to measurement errors, or noise.

Keywords: Partial Differential Equations , linear equations, errors.

1. Introduction

1.1 Preliminary considerations

Partial differential equations model phenomena in physics, engineering, biology, chemistry, physics.

nances and other sciences. The parameters of these problems are, in many cases, the coefficients that appear in the Partial Differential Equations that model these problems. For linear equations homogeneous, it is theoretically possible to find these coefficients from a group of solutions of these EDPs. In a practical situation in which we want to know the coefficients of an EDP to from observations of the phenomenon it models, which characterizes an inverse problem, the solutionstions observed would not be exact, they would be an approximation, possibly with measurement errors, or noise, of exact solutions. In this way we will only get an estimate of the coefficients.

Thus, we need to know the behavior of the estimators used when applied to these approximations and in relation to measurement errors, or noise

In this work we will analyze the estimators proposed in [11], for the functional coefficients of a homogeneous linear EDP.

Estimators

We will now present the estimators with integrals, which are the object of study of this work, and the direct estimators that will be used for comparison. Both groups of estimators estimate the functional parameters of an EDP of the following type:

$$f\frac{\partial^2 u}{\partial t^2} + g\frac{\partial u}{\partial t} + hu = \frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right), \quad (1)$$

where the parameters f, g, h and k are functions that depend only on x. This EDP has as casesnparticular the heat and wave equations with functional coefficients that are used for the modeling of heat and wave phenomena. Both groups of estimators assume the existence a set of functions with at least four functions, which represent EDP solutions (1).

The functions are of the form

$v = u + \varepsilon$

where u is an EDP solution and is an uncorrelated noise.The description of the estimators with integrals was based on the work

Estimators with integrals

In estimators with integrals, we assume that we have N functions available V_1 (x, t), · · · , V_N (x, t)

where V_i = u_i+$\varepsilon_i$for all i , 1 ≤i ≤ N where u_i is an EDP solution and ε_i is the uncorrelated noise. that satisfies a condition in the frequency domain. The V_i functions are well known but we have no information about the solutions u_i nor about the realizations of random noises ε_i . Atvariables x and t belong to the intervals [0, L] and [0, T] respectively. Based on these assumptionsnwe will obtain consistent estimators $\hat{f}$, $\hat{g}$, $\hat{h}$, and $\hat{k}$,, for f, g, h and k.

getting the estimator

To simplify the presentation, let's assume that the number of available functions is one multiple of four, N = 4n, and let's divide the functions into four sets with n elements, I_1= {V_1, … . , V_n }

$$I_2 = \{v_{n+1}, \cdots, v_{2n}\},\ I_3 = \{v_{2n+1}, \cdots, v_{3n}\}$$

$$I_4 = \{v_{3n+1}, \cdots, v_{4n}\}$$

for example,Let λ and μ be real numbers in the range of (0, 1).Partially integrating with respect to the second variable over the interval [λT − μλt, λT + μ (1 − λ)t] followed by integration with respect to t over [0, T] and integrating into relation to (λ, μ) ∈ $(0, 1)^2$ from equation (1) gives:

$$f(x)\int_0^T u(x,t)\,Q(t)\,dt$$

+

$$g(x)\left(\int_0^T u(x,t)\,R(t)\,dt\right)$$

+

$$h(x)\int_0^T u(x,t)\,S(t)\,dt = \frac{\partial}{\partial x}\left[k(x)\frac{\partial}{\partial x}\int_0^T u(x,t)\,S(t)\,dt\right]$$

Where

$$Q(t) = \frac{6t^2 - 6Tt + T^2}{6T^3}$$

$$R(t) = \frac{-2t^3 + 3t^2T - tT^2}{6T^3}$$

$$S(t) = \frac{(T-t)^2\,t^2}{12T^3}$$

Denoting

$$\bar{u}^j = \frac{\sum_{i\in I_j} u_i}{n},$$

due to the linearity of the partial integro-differential equation (EIDP) above, we can write,

for 1 ≤ j ≤ 4,

$$f(x)\int_0^T \bar{u}^j(x,t)\,Q(t)\,dt$$

+

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

$$g\left(x\right)\left(\int_0^T \bar{u}^j\left(x,t\right)R\left(t\right)dt\right)$$

+

$$h\left(x\right)\int_0^T \bar{u}^j\left(x,t\right)S\left(t\right)dt =$$

$$\frac{\partial}{\partial x}\left[k\left(x\right)\frac{\partial}{\partial x}\int_0^T \bar{u}^j\left(x,t\right)S\left(t\right)dt\right] \quad (3)$$

Be

$$\alpha\left(\bar{u}^j, x\right) = \alpha_j = \int_0^T \bar{u}^j\left(x,t\right)Q\left(t\right)dt \quad (4)$$

$$\beta\left(\bar{u}^j, x\right) = \beta_j = \int_0^T \bar{u}^j\left(x,t\right)R\left(t\right)dt \quad (5)$$

And

$$\gamma\left(\bar{u}^j, x\right) = \gamma_j = \int_0^T \bar{u}^j\left(x,t\right)S\left(t\right)dt \quad (6)$$

Thus, we can write the system of four equations

$$\alpha_j\left(x\right)f\left(x\right) + \beta_j\left(x\right)g\left(x\right) + \gamma_j\left(x\right)h\left(x\right)$$

$$= \frac{\partial}{\partial x}\left[k\left(x\right)\frac{\partial}{\partial x}\gamma_j\right]$$

$$= k'\left(x\right)\gamma_j' + k\left(x\right)\gamma_j'' \quad (7)$$

ISSN: 2776-0960 Volume 3, Issue 10 Oct., 2022

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

Denoting

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4),$$

$$\beta = (\beta 1, \beta_2, \beta_3, \beta_4),$$

$$\gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4),$$

$$\left(\gamma_1^{'}, \gamma_2^{'}, \gamma_3^{'}, \gamma_4^{'}\right)$$

$$\gamma^{''} = \left(\gamma_1^{''}, \gamma_2^{''}, \gamma_3^{''}, \gamma_4^{''}\right),$$

Where

$$\gamma_i^{'} = fracd\gamma_i dx \text{ e } \gamma_i^{''} = fracd^2\gamma_i dx^2,$$

$$i = 1, 2, 3, 4$$

Solving the system we get

$$k(z) = k(0)\exp\left(-\int_0^z \frac{\det(\alpha, \beta, \gamma, \gamma'')}{\det(\alpha, \beta, \gamma, \gamma')} dx\right) \quad (8)$$

$$f(x) = k(x)\frac{\det(\gamma'', \beta, \gamma, \gamma')}{\det(\alpha, \beta, \gamma, \gamma')} \quad (9)$$

$$g(x) = k(x)\frac{\det(\alpha, \gamma'', \gamma, \gamma')}{\det(\alpha, \beta, \gamma, \gamma')} \quad (10)$$

$$h(x) = k(x)\frac{\det(\alpha, \beta, \gamma'', \gamma')}{\det(\alpha, \beta, \gamma, \gamma')} \quad (11)$$

More briefly:

$$k = K(\alpha, \beta, \gamma) = K^*\left(\bar{u}^1, \bar{u}^2, \bar{u}^3, \bar{u}^4\right)$$
$$f = F(\alpha, \beta, \gamma) = F^*\left(\bar{u}^1, \bar{u}^2, \bar{u}^3, \bar{u}^4\right)$$
$$g = G(\alpha, \beta, \gamma) = G^*\left(\bar{u}^1, \bar{u}^2, \bar{u}^3, \bar{u}^4\right)$$
$$h = H(\alpha, \beta, \gamma) = H^*\left(\bar{u}^1, \bar{u}^2, \bar{u}^3, \bar{u}^4\right)$$

denoting

$$\bar{v}^i = \frac{\sum_{i \in I_j} v_i}{n},$$

Saw , for 1 ≤ j ≤ 4, defining $\hat{\alpha}_j, \hat{\beta}_j, \hat{\gamma}_j$

by replacing $\bar{u}^j$ for $\bar{v}^j$

equations (4), (5) and (6), and using vector notation.

We now define our estimators for the functional coefficients as

$$k = K\left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right) = K^*\left(\bar{v}^1, \bar{v}^2, \bar{v}^3, \bar{v}^4\right)$$

$$f = F\left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right) = F^*\left(\bar{v}^1, \bar{v}^2, \bar{v}^3, \bar{v}^4\right)$$

$$g = G\left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right) = G^*\left(\bar{v}^1, \bar{v}^2, \bar{v}^3, \bar{v}^4\right)$$

$$h = H\left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right) = H^*\left(\bar{v}^1, \bar{v}^2, \bar{v}^3, \bar{v}^4\right)$$

We observe that the model (1) is unidentifiable as can be clearly seen by the presence

of the constant k(0) in (8), and, consequently, in (9), (10) and (11). However, if, for

For example, we know that

$$\int_0^L k(x)\,dx = y$$

so from (7) we have that

$$k(0) = y\left[\int_0^L \exp\left(\frac{\det\left(\alpha, \beta, \gamma, \gamma''\right)}{\det\left(\alpha, \beta, \gamma, \gamma'\right)} dx\right)\right]^{-1}$$

and identifiability is guaranteed.

Main Results

Let's assume that noise has a frequency domain representation. Using the notation complex we write

$$\epsilon_i(x,t) = \sum_{l,m \in \mathbb{Z}} a_{i,l,m} e^{l\left(\frac{2\pi l x}{L} + \frac{2\pi m t}{T}\right)}$$

ISSN: 2776-0960 Volume 3, Issue 10 Oct., 2022

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

From now on, let's assume that the noise satisfies the following conditions:

1. for all i, l and m

$$\mathbb{E}a_{i,l,m} = 0;$$

2. for all i, j, l, m, p and q such that

$(i,l,m) \neq (j,p,q)$, $\mathbb{C}ov\left(a_{i,l,m}, a_{j,p,q}\right) = 0;$

3. $\frac{1}{n^2}\sum_{i=1}^{n}\sum_{l,m\in\mathbb{Z}} l^4\mathbb{E}\left(a_{i,l,m}^2\right) \to 0$

when n → ∞

Clearly, condition [3] is fulfilled in the case where noise is identically distributed (which

means that, in the frequency domain, for all i and j and for all l and m, we have

$a_{i,l,m} = a_{j,p,q}$

And satisfies P

$\sum_{l,m\in\mathbb{Z}} l^4\mathbb{E}\left(a_{i,l,m}^2\right).$

Condition [2] imposes the non-correlation of the inter and within the noise. In this case, we have the independent noises in pairs, that is,

$\epsilon_i \perp \epsilon_j$, for $i \neq j$

we need the fulfillment of

$\mathbb{C}ov\left(a_{i,l,m}, a_{j,p,q}\right) = 0,$

Cov (ai,l,m, aj,p,q) = 0 in order to obey [2]. This is also the case.

if we have independent noises.

Note that Theorem 1, Corollary 1 and Theorem 2 concern the form of estimation. He can-

Let us assume an arbitrary non-zero value for

$\widehat{K}(0)$ = k (0) or consider the representatives

class patterns, to pair the functions. Using the notation established so far, we can state the following:

Theorem 1. Let the functional coefficients f, g, h and k that appear in (1) be such that (f, g, h, k) ≠
(0, 0, 0, 0) and, for all x ∈ [0, L],

$$\det\left(\alpha,\beta,\gamma,\gamma'\right)\neq 0.$$

Assuming that the noises satisfy the conditions
tions 1, 2 and 3 above. Then, the integrated mean of the squared errors

$$\mathbb{E}\left(\left\|\hat{k}-k\right\|_2^2\right),\ \mathbb{E}\left(\left\|\hat{f}-f\right\|_2^2\right),$$

$$\mathbb{E}\left(\|\hat{g}-g\|_2^2\right)\ \ \mathbb{E}\left(\left\|\hat{h}-h\right\|_2^2\right)$$

tend to zero when N −→ ∞.
This result is independent of the value of k(0). the condition of

$$\det\left(\alpha,\beta,\gamma,\gamma'\right)\neq 0$$

implies that

$$\{\hat{u}_1,\hat{u}_2,\hat{u}_3,\hat{u}_4\}$$

are linearly independent.

Corollary 1. Under the same assumptions as in Theorem 1, the estimators ˆ$\hat{f}$, $\hat{g}$, $\hat{h}$, and
$\hat{k}$, are consistent, Theorem 2. Under the assumptions of Theorem 1 and on the assumption that noise is i.i.d. like, we have mos

$$\sqrt{N}\left(\hat{f}-f\right)\rightsquigarrow T_f(v),\ \sqrt{N}\left(\hat{g}-g\right)\rightsquigarrow T_g(v)$$

,

$$\sqrt{N}\left(\hat{h}-h\right)\rightsquigarrow T_h(v)$$

And

$$\sqrt{N}\left(\hat{k}-k\right)\rightsquigarrow T_k(v),$$

Where

$$T_f(v) = F^{*'}\left(\hat{u}^1, \hat{u}^2, \hat{u}^3, \hat{u}^4(v_1, v_2, v_3, v_4)\right),$$
$$T_g(v) = G^{*'}\left(\hat{u}^1, \hat{u}^2, \hat{u}^3, \hat{u}^4(v_1, v_2, v_3, v_4)\right),$$
$$T_h(v) = H^{*'}\left(\hat{u}^1, \hat{u}^2, \hat{u}^3, \hat{u}^4(v_1, v_2, v_3, v_4)\right),$$
$$T_j(v) = K^{*'}\left(\hat{u}^1, \hat{u}^2, \hat{u}^3, \hat{u}^4(v_1, v_2, v_3, v_4)\right).$$

Here $\{V_1, V_2, V_3, V_4\}$ are i.i.d. with distribution identical to that of v and v has given spectral representation

$$v(x,t) = \sum_{l,m\in\mathbb{Z}} b_{l,m} e^{l\left(\frac{2\pi l x}{L} + \frac{2\pi m t}{T}\right)}$$

with uncorrelated coefficients

$$b_{l,m} \sim N\left(0, \sigma^2_{l,m}\right),$$

where ${\sigma^2}_{l,m}$ is the common variance of $a_{i,l,m}$

3 Observations

It is observed that the methodology presented here, that is, the integration of EDP in a consensibly chosen, the analytical solution of the new integrated EIDP for the functional coefficients and, finally, the replacement of true EDP solutions by averages of the solutions measured in the analytic expressions for the functional coefficients, can be applied to a larger class of EDPs that include EDPs of the type and other types.

$$\sum_{i=0}^{m} f_i \frac{\partial}{\partial x}\left[k\frac{\partial u}{\partial x}\right]$$

We can define other estimators for f, g, h and k by first smoothing out our functional data. This can be done, for example, simply by vi smoothing , using a low-pass filter and then using the smoothed versions of vi in the estimators' equations. We hope this procedure gives good results in case of high frequency noise. The conditions we assume in relation to behavior in the frequency domain of noise can be relaxed, more specifically, there may be some moderatecorrelation between the coefficients Fourier entities, both inter and intra noise and the consistency of the estimators will still be maintained.

Note that although the estimators $\hat{f}$, $\hat{g}$, $\hat{h}$, and $\hat{k}$, are consistent, they are, in principle, dependent on teeth of choice of sets I_1, I_2,, I_3, I_4,.

2 Direct estimators

The direct estimators also use four functions $\{V_1, V_2, V_3$ and $V_4\}$ associated with EDP (1)

where $V_i = u_i + \epsilon_i$

defining

$$U(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t), u_4(x,t)),$$

Be

$$\frac{\partial^2 U}{\partial t^2}(x,t) =$$

$$\left(\frac{\partial^2 u_1}{\partial t^2}(x,t), \frac{\partial^2 u_2}{\partial t^2}(x,t), \frac{\partial^2 u_3}{\partial t^2}(x,t), \frac{\partial^2 u_4}{\partial t^2}(x,t)\right),$$

$$\frac{\partial U}{\partial t}(x,t) =$$

$$\left(\frac{\partial u_1}{\partial t}(x,t), \frac{\partial u_2}{\partial t}(x,t), \frac{\partial u_3}{\partial t}(x,t), \frac{\partial u_4}{\partial t}(x,t)\right),$$

$$\frac{\partial^2 U}{\partial x^2}(x,t) =$$

$$\left(\frac{\partial^2 u_1}{\partial x^2}(x,t), \frac{\partial^2 u_2}{\partial x^2}(x,t), \frac{\partial^2 u_3}{\partial x^2}(x,t), \frac{\partial^2 u_4}{\partial x^2}(x,t)\right),$$

$$\frac{\partial U}{\partial x}(x,t) =$$

$$\left(\frac{\partial u_1}{\partial x}(x,t), \frac{\partial u_2}{\partial x}(x,t), \frac{\partial u_3}{\partial x}(x,t), \frac{\partial u_4}{\partial x}(x,t)\right).$$

The parameters f, g, h, and k can be obtained by the equalities (12):

$$k(z) = k(0)\exp\left(-\int_0^z \frac{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, U, \frac{\partial^2 U}{\partial x^2}\right)}{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, U, \frac{\partial U}{\partial x}\right)} dx\right),$$

$$f(x) = k(x)\frac{\det\left(\frac{\partial^2 U}{\partial x^2}, \frac{\partial U}{\partial t}, U, \frac{\partial U}{\partial x}\right)}{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, U, \frac{\partial U}{\partial x}\right)},$$

(13)

$$g(x) = k(x)\frac{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial^2 U}{\partial x^2}, U, \frac{\partial U}{\partial x}\right)}{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, U, \frac{\partial U}{\partial x}\right)},$$

(14)

$$h(x) = k(x)\frac{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, \frac{\partial^2 U}{\partial x^2}, \frac{\partial U}{\partial x}\right)}{\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, U, \frac{\partial U}{\partial x}\right)},$$ (15)

and the direct estimators $\tilde{f}$, $\tilde{g}$, $\tilde{h}$ and
$\tilde{k}$ are defined by replacing u by v in equations (12) and (15).

We should note that the parameters f, g, h and k depend only on x. Thus, they are estimated
for an arbitrary value of t ∈ [0, T]. Theoretically, the parameter estimates are independent
of the chosen value of t.

Numerical solutions

For the equation:

$$f\frac{\partial^2 u}{\partial t^2} + g\frac{\partial u}{\partial t} + hu = \frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right)$$

- $f\left(x\right)=\frac{x^4+x^3+x^2+x+1}{3}$
  - $g\left(x\right)=\frac{x^3+x^2+x+1}{3}$
  - $h\left(x\right)=\frac{x+1}{3}$
  - $k\left(x\right)=\frac{x^4+x^3+x^2+x+1}{3}$,

To determine the numerical solutions of equation it is necessary to impose initial conditions and
outline. As the equation is of the second degree in t and x we must impose two conditions for each
variable. We apply the following initial and boundary conditions.

$$u\left(x,0\right) = l_1\left(x\right)$$

$$\frac{\partial u(x,0)}{\partial t} = l_2\left(x\right)$$

$$\frac{\partial u(0,t)}{\partial x} = m_1\left(t\right)$$

$$\frac{\partial u(L,t)}{\partial x} = m_2\left(t\right).$$

First, equation (1) is decomposed into a system of equations with first derivatives.

$$\begin{cases} \frac{\partial u}{\partial t}=v \\ \frac{\partial v}{\partial t}=\frac{1}{f(x)}\left(-g\left(x\right)v-h\left(x\right)u+\frac{\partial}{\partial x}\left(k\left(x\right)\frac{\partial u}{\partial x}\right)\right) \end{cases}$$

Expanding the derivatives in x we have

$$\begin{cases} \frac{\partial u}{\partial t}=v \\ \frac{\partial v}{\partial t}=\frac{1}{f(x)}\left(-g\left(x\right)v-h\left(x\right)u+k'\left(x\right)\frac{\partial u}{\partial x}+k\left(x\right)\frac{\partial^2 u}{\partial x^2}\right) \end{cases}$$

Simulation procedure

In this section we will describe the procedure used in the estimation simulations of the coefficients

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

EDP functionalities (1). The procedure was developed in the statistical program R, version 3.2.3,

and using the deSolve, RandomFields and pracma utility packages.

We selected EDPs, which are particular cases of EDP (1), to apply the estimators in the

solutions of these equations.

To measure the performance of the estimators, we will use the quadratic errors described below.

$$\left\|\hat{f}-f\right\|_2^2=\int_0^L\left(\hat{f}(x)-f(x)\right)^2 dx$$

$$\|\hat{g}-g\|_2^2=\int_0^L(\hat{g}(x)-g(x))^2 dx$$

$$\left\|\hat{h}-h\right\|_2^2=\int_0^L\left(\hat{h}(x)-h(x)\right)^2 dx$$

$$\left\|\hat{k}-k\right\|_2^2=\int_0^L\left(\hat{k}(x)-k(x)\right)^2 dx$$

Selection of solutions

Analyzing the expressions of the two groups of estimators we see that there may be a problem of

division by zero. From expressions (8) to (11) we see that if the determinant.

$\det(\alpha,\beta,\gamma,\gamma')$

results in zero for some point in the interval [0, L] division by zero will occur. in the estimators

direct we will have the same problem for the determinant.

$$\det\left(\frac{\partial^2 U}{\partial t^2},\frac{\partial U}{\partial t},U,\frac{\partial U}{\partial x}\right)$$

in equation it was possible to apply the estimators, although the determinants vanish in

a point of the interval [0, 1], as you can see in graphs 1. Below are the graphs for the

equation.

ISSN: 2776-0960 Volume 3, Issue 10 Oct., 2022

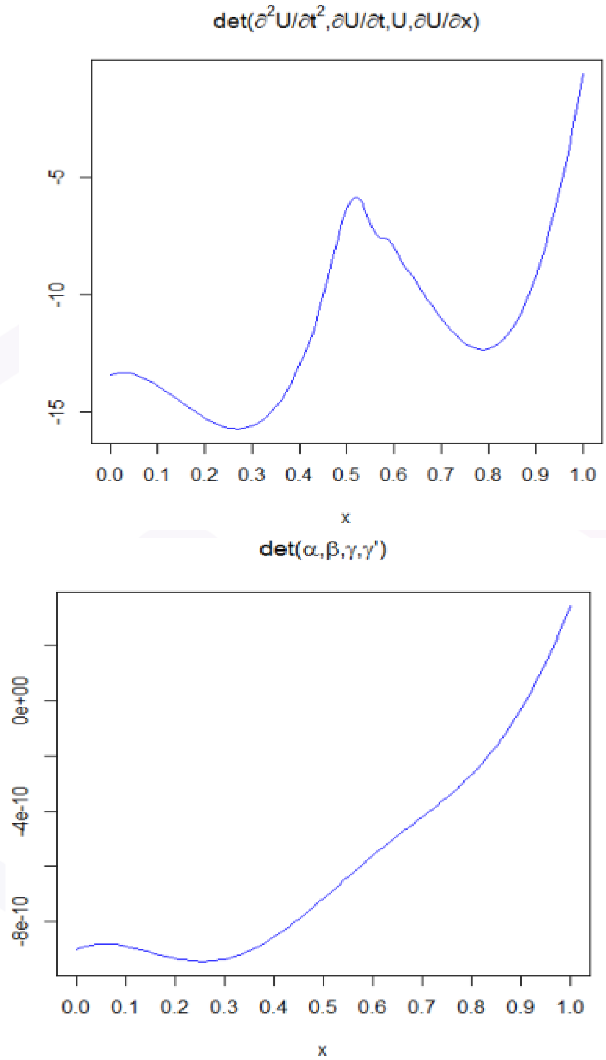
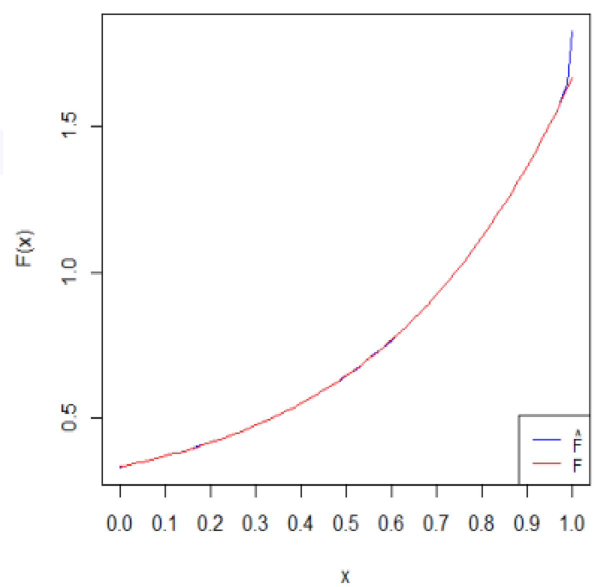


Figure 1: Determinant used in the equation estimators



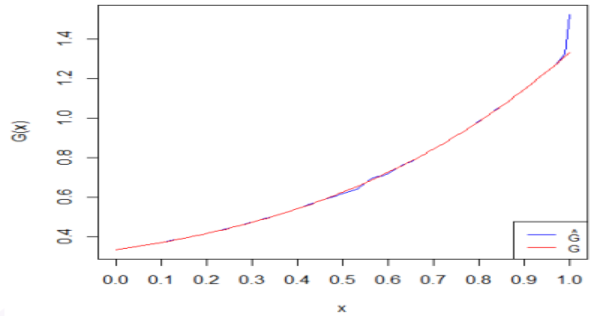
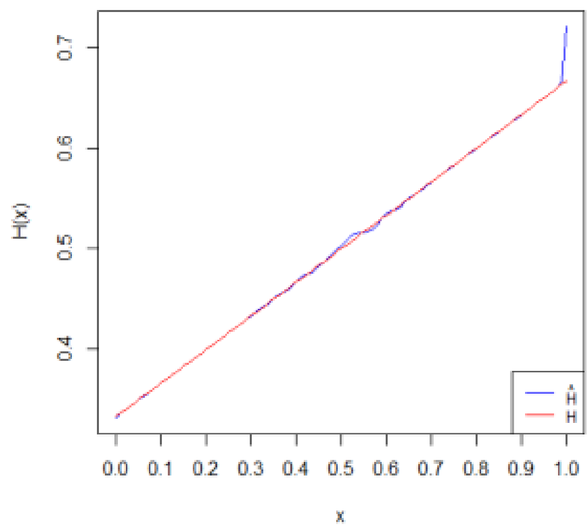


Figure 2: Estimation of F and G functions of the equation.



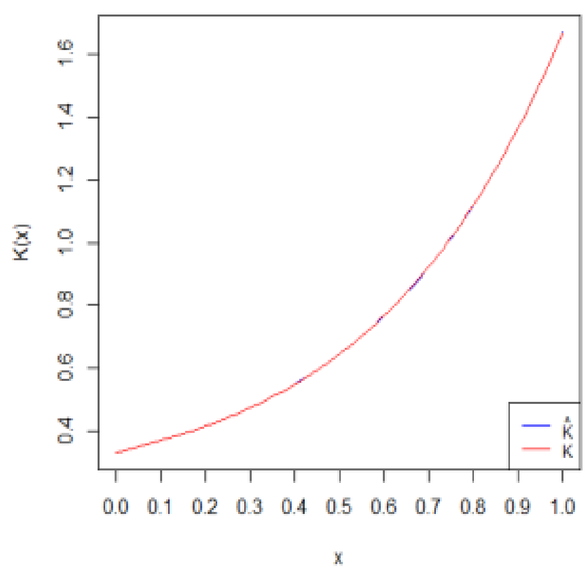


Figure 2: Estimation of h and k functions of the equation.

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org

3. Results & Discussion

Thus, we see that the selection of the solutions used has an impact on the estimation of the coefficients. you Even if the solutions are linearly independent, the determinants.

$$\det(\alpha, \beta, \gamma, \gamma')$$

And

$$\det\left(\frac{\partial^2 U}{\partial t^2}, \frac{\partial U}{\partial t}, U, \frac{\hat{\partial U}}{\partial x}\right)$$

may be null at some point in the interval [0, L], and make the application unfeasible

estimators or generate very bad estimates.

4. Conclusions

In this work we evaluate the performance of the estimators, with integrals, for the coefficients EDP functionalities (1). We have seen that in the application of the estimators, division by zero can occur when the determinant det (α, β, γ, γ0)is equal to zero. This problem can be circumvented by choosing a different set of solutions, used in the application of the estimators. we observe that estimators with integrals use all the information contained in the solutions, whereas estimators with direct uses information relative to a fixed value of t. This makes the performance of direct estimators is dependent on the choice of t. Comparing the two groups of estimators we have seen that for noiseless solutions the estimators have equivalent performance. already in the presence of noise in the solutions, the performance of estimators with integrals is superior to that of direct ones. So we see that the estimators with integrals present better characteristics than the estimators direct.

References

1. Victor A Bloomfield. Using R for Numerical Analysis in Science and Engineering. CRC Press, 1st edition, 2014. 41, 42.
2. Karline Soetaert; Jeff Cash and Francesca Mazzia. Solving Differential Equations in R. Spring ger, 1st edition, 2012. 41.

3. Jose C S de Miranda. Estimation of functional coefficients in partial differential equations. Physia-Verlag, 2011. 1, 2, 25.
4. Andrei D Polyanin. Handbook of Linear Partial Differential Equations for Engineers and Scientists. CHAPMAN and HALL/CRC, 1st edition, 2002. 14, 31.
5. Y Pinchover and J Rubenstein. An Introduction to Partial Differential Equations. Cambridge University Press, 1st edition, 2005. 14, 15
6. Karline Soetaert; Thomas Petzold and R Woodrow Setzer. Solving differential equations in a: Package desolves. Journal of Statistical Software, 33:1–25, 2010. 41
7. William E Schiesser. The Numerical Method of Lines: Integration of Partial Differential Equations. Academic Press, 1st edition, 1991. 15, 40.
8. William E Schiesser and Graham W Griffiths. A Compendium of Partial Differential Equation Models - Method of Lines Analysis with Matlab. Cambridge University Press, 1st edition, 2009. 15, 40.
9. Walter A Strauss. Partial Differential Equations: An Introduction. Springer, 2nd edition, 2008. 14
10. JW Thomas. Numerical Partial Differential Equations: Finite Difference Methods. Spri- ger Verlag, 1 edition, 1995. 15
11. Andrew T A Wood and Grace Chan. Simulation of stationary gaussian processes in [0, 1]2 Journal of Computational and Graphical Statistics, 3:409–432, 2010. 23.

ResearchJet Journal of Analysis and Inventions
https://reserchjet.academiascience.org