AN INTELLIGENT SOLUTION TO PARTIAL DIFFERENTIAL EQUATIONS USING NEURAL NETWORKS

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Abstract

As a powerful information processing tool, neural network has been widely used in the fields of computer vision, biomedicine, and oil and gas engineering.

Technological changes in the field. Deep learning networks have very strong learning capabilities, not only to discover physical laws, but also to solve partial differential equations. In recent years, based on

The solution of partial differential equations in deep learning has become a new research hotspot. Following the terms of traditional analytical solutions of partial differential equations and numerical solutions of partial differential equations, this paper calls

The method of solving partial differential equations with neural network is the intelligent solving method of partial differential equations or the neural network solving method of partial differential equations.

First, the development process of intelligent solution of partial differential equations is briefly introduced, and then it is expanded from the perspectives of inverting unknown partial differential equations and solving known partial differential equations.

Discussion, focusing on solving methods of known partial differential equations .

According to the construction method of the loss function in the neural

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network, the partial differential equation solving methods are divided into There are three categories: the first category is data-driven, which mainly learns partial differential equations from data, which can be applied to recovery equations, parameter inversion, etc.; the second category is physical Reasonable constraints, that is, on the basis of data-driven, supplemented by physical constraints, adding physical laws such as control equations to the loss function, reducing the network's impact on label data

Dependence on the generalization ability and application value; the third type of physics-driven (pure physical constraints), does not use label data at all, only through the laws of physics

Solving partial differential equations is currently only applicable to simple partial differential equations. This paper introduces the research progress of intelligent solving of partial differential equations from these three aspects.

and fully connected neural network, convolutional neural network, cyclic neural network and other network structures. Finally, the research progress of intelligent solution of partial differential equations is summarized.

The corresponding application scenarios and future research prospects are given.

Key words: neural network, intelligent solving of partial differential equations, data-driven, physical constraints

1. **Introduction**

Artificial intelligence has triggered technological changes in many fields and is widely used in computing

Computer vision, biomedicine, oil and gas engineering development and other fields. Deep Science Deep learning in Engineering Technology,Fluid Mechanics, Computational MechanicsResearch in other fields has important theoretical guiding significance and engineering application. value. In recent years, based on the dynamic and static data of reservoirs, artificial intelligence has It is hoped to realize the fine description and precise development of the reservoir, and improve the oi1 recovery.

Intelligent integration of well, fracturing construction, production data, etc.,

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greatly improves Improve fracturing effect and reduce development cost. Big data and intelligent optimization The combination of methods will transform oilfield data analysis methods, oilfield development Control and optimization methods [1] Unconventional oil and gas development problems and artificial intelligence

It is expected to solve the establishment of unconventional complex oil and gas physical laws,

Problems such as solving partial differential equations. Artificial intelligence and big data will "realize

The grand goal of upgrading the main technology of petroleum exploration and development, from the perspective of technology

Promote the overall transformation and upgrading of the petroleum exploration and development industry at the technical level" Artificial intelligence methods are outstanding due to their outstanding ability to deal with highly complex problems. It has attracted special attention in the field of oilfields[1] Traditional artificial nerve Networks have been widely used in petroleum engineering, such a predicting future Well logging data of known years[3], predicted oil pressure-volume-temperature properties properties[4], predicted production profile[5], estimated porosity[6], bottom hole flowing pressure[7], selection of completion methods for shale gas reservoirs [8], well testing interpretation [7], etc.Deep Learning is a new field of Machine Learning. Deep LearningThe essence of it is to build a network model with multiple hidden layers, through learning Learning large-scale data to obtain more representative features, thereby improving Accuracy of prediction and classification. Tian and Horne [8] use recurrent neural E-learning permanent downhole pressure gauge (PDG) data for oil identification Tibetan model and production prediction. Sudakov et al. [9] used deep learning for Permeability prediction. Mosser et a1. [11] used deep learning for 3D multi-dimensional Reconstruction of porous medium. Zhang Dongxiao et al. [7] used recurrent neural network to study measurement Generation and repair of well curves. In the past two years, deep learning has Automatic inversion has been well applied [6]

At the same time, in solving parameter inversion, digital core, logging curve

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On issues such as line, well test interpretation, etc., deep learning as an artificial intelligence development
The exhibition engine has excellent performance[5]
with deep learning as the core
Artificial intelligence is setting off a new research boom in the field of oil and gas development.
The most forward-looking and disruptive research in the
Solving partial differential equations. Once this method is broken through, the laws of physics are established,
Both parameter inversion and numerical simulation methods will undergo changes, and my country will also
In industrial computing centered on the solution of partial differential equations (partial differential equations)
There is a huge opportunity in computing software. Since 2017, deep learning has Discovery of physical laws, inversion of reservoir parameters and solving of partial differential equations
Played a surprising role [3]
In this paper, the deep learning representation partial differential equation is divided into two fields
Scene: Constructing Unknown Partial Differential Equations and Solving Known Partial Differential Equations. Yes
In order to construct unknown partial differential equations, this paper briefly introduces the network structure
Intrinsic connection with partial differential equations, differential operators or evolution operators, etc.
Department, outlines the representation method of neural network approximation to unknown partial differential equations, And give the problems and difficulties to be solved. For the solution known Partial Differential Equations, this paper focuses on data driven, physical constraints and physical driven Introduce the neural network to solve the known partial differential equations from three angles method, mainly including the principle of neural network solving partial differential equations, network box framework construction, loss function construction, etc., combined with research status at home and abroad, the Department of systematically sort out the research context in this field,

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and analyze the partiality of neural network solutions.
The key problems and solutions existing in the sub-equation, and the feasible
Future research directions and contents are discussed and prospected. In addition, although deep
Degree learning has developed rapidly in recent years, but it is still difficult to solve. The research on mechanical problems such as partial equations is still limited, and the practical application
The performance in the progress on.

1 **Discussion on the method of solving** partial differential equations based on neural network research In 1943 McCulloch and Pitts [4] established neural networks and its mathematical model, ushering in a new era of artificial neural network research. The back-propagation algorithm was first proposed in the mid-1980s
and its development [5] led to the second study in the field of artificial neural networksupsurge.
For a long time, people have hoped to find

new numerical method for solving partial differential equations for nonlinear equations. Its
One of the explorations is the solution method based on artificial neural network.
Automatic
automatic differentiation can be exact using the chain rule
Calculate Derivatives [8]
, according to the input coordinates of the neural network and the network
Differentiate the entire neural network model by using the network parameters to replace the partial
Complex gradient calculations in differential equations, based on artificial neural networks
The solution of partial differential equations has laid the foundation.
In the 1990s, some scholars began to study the use of God
Mathematical Foundations and Methods for Solving Differential Equations via Networks. 1990

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Wornik et al. [9] proved that a multilayer neural network can approximate any function.
numbers and their derivatives. This lays the groundwork for the neural network solution of differential equations
Theoretical basis. Subsequently, Li [5] proved a neural network with hidden layers Approximate multivariate polynomial functions and their derivatives. Lagaris et al.
[l] put the differential
The initial value and boundary conditions in the partial equation are independently characterized, and it is quite new to propose Ying's method for solving partial differential equations. Subsequently, many scholars have explored
For example, Aarts and Van [2] will characterize different order differential arithmetic
The single-hidden layer feed-forward networks of the sub-layers are combined and jointly trained to solve the partial Differential equations; another example, Ramuhalli et a1. [3] embedded the finite element model
into the neural network, the finite element neural network is proposed.
The limitations of the layer feedforward neural network model, the early methods can only solve
Simple partial differential equations, partial differential equations based on neural network
The solution method has not received enough attention.
Early methods were mainly based on data- driven, i.e. obtaining partial
the input and exact solution of the sub- equation (often called "labeled data"), then Approximate the labeled data with a neural network to obtain
The neural network model of the division equation, as shown in Figure 1(a). The input of the network
It can be parameters or space, time, etc., which can be selected according to needs.

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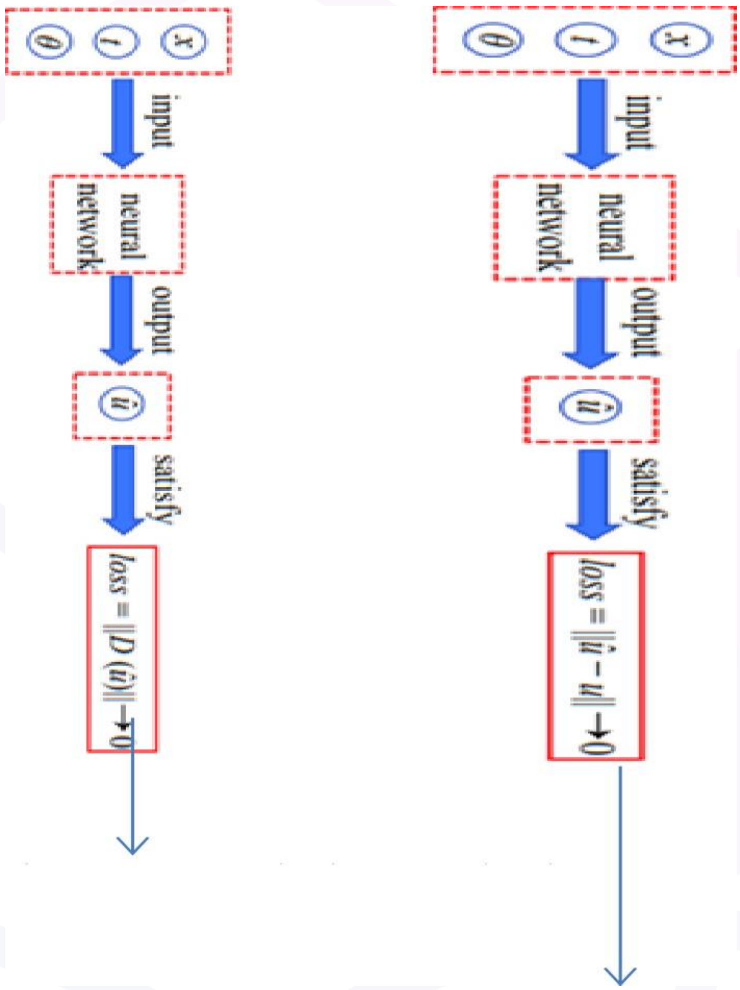
With the successful application of deep learning algorithms in many fields



Data-driven PDE solution method the exact solution is the label.

Physics-driven PDE solution method without any labeled data .
Scholars at home and abroad have reopened partial differential equations based on neural networks
Research on solving methods, made a series of breakthroughs, and proposed new methods, such as
A purely physics-driven method for solving partial differential equations.

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The method uses the control
Equations are constrained without labeled data, as shown in Figure 1(b).

According to different application scenarios, this paper will invert from deep learning Constructing unknown partial differential equation and solving known partial differential equations The next section mainly introduces how to approximate by neural network.
Linear or nonlinear operators to find hidden partial differentials from data process model.

2 Inversion of unknown partial differential equations based on deep learning

Using deep learning methods to invert unknown partial differential equations from data
is one of the current research hotspots. For unknown partial differential equations, the main
The main research goal is to find out what lies behind the data through deep learning Partial differential equation model, inversion of unknown partial differential equations from data
(such as the right-hand term of the equation, the integral form of the equation, or the evolution of the equation
operator, etc.), and further build the model for solving.
The traditional way of restoring equations is to construct simple functions and partial derivatives
Alternative dictionaries for . These functions and partial derivatives are likely to appear in unknown
in the governing equation of . The nonlinear response according to the known partial differential equation
The model is constructed in the form of , and then the sparse regression class method is used to learn These unknown parameters, select the term that most accurately represents the data. This traditional The recovery method requires the assumption that the nonlinear response form is known or the micro-
The method of finite difference approximation of molecular operators, and deep learning greatly reduces the

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The requirement for prior knowledge of partial differential equations is reduced, and only simple prior knowledge, such as the largest possible order of the equation. Also, sparse regression methods The numerical approximation of the spatial difference in the dictionary needs to be determined in advance, which limits the

The expressive and predictive ability of the dictionary and the need to build a sufficiently large dictionary, which may lead to high memory load and computational cost, especially Especially when the number of model variables is large. Deep learning methods Using a learnable convolution approximate differential operator or approximate evolution operator, Fundamentally improve the ability to identify dynamics from noisy data, from And make the model have stronger expression ability and prediction accuracy. Such as Without sufficient knowledge of the data, it is also possible to adjust the polynomialThe differential of the neural network to obtain a better representation effect, the neural network in the partial differential There is a lot to do in equation solving and recovery problems.

In recent years, scholars at home and abroad have devoted themselves to exploring the network structure and bias.

Differential equations, differential operators or evolution operators of equations, etc.

In the connection, theoretically supports the use of deep learning to characterize partial differential equations

Cheng. In 2018, Long et a1. [6] proposed a

data-driven

Feedforward neural network (PDE-Net), its core idea is: time derivative

The term is Euler discrete, the constrained convolution kernel approximates the differential operator, and then makes.

Intrinsic correlation [60-61]

According to this association, Long et a1.

[5] proposed a constrained

The convolution kernel, that is, it can be proved mathematically that the convolution can represent the differential operator, Therefore, the convolution kernel is limited. Deep learning learns on a limited basis

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Convolution kernels areused to learn convolution kernels, so as to have better representation ability of partial differential operators. For example,

For convolution kernel

$$a_{0,1} = \frac{1}{4}\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}$$

Apply the convolution kernel to

On two-dimensional space variables, we get

$$\frac{2}{\quad}\text{-(g()}_{jj}\ {}_{\text{I}}[-\cdot]\otimes u)[i,j] = \frac{1}{2h}(u(x_i,y_j)-u(x_i,y_j-h))+$$

- Jj))

$$\frac{1}{2h}(u(x_i-h,y_j)-u(x_i-h,v_j$$

-' il,l.f,!'jj z\ li + 0.

Similar to the order of vanishing moments in wavelet theory, for the convolution kernel ,

Define the order of the vanishing moment of

Z• $q\backslash k\backslash$—— 0

$k\in Z^2$ In § = (fii. 2)* z² for satisfying

or · · $|\beta| < |\alpha|$

|Q| = |a| where |Q| |a Taylor expansion to derive

$$\frac{1}{\varepsilon^{|\alpha|}}\sum_{k\in Z^2} q[k]F(x+k\varepsilon) = C_\alpha \frac{\partial^\alpha F(x)}{\partial x^\alpha} + o(\varepsilon),$$

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And

$$C_\alpha = \sum_{k \in Z^2} k^\alpha q[k]$$

$\alpha!$

Finally, the convolution kernel can be constructed by formula (2) to approximate the differential operator,

See [5] for details. Based on the constrained convolution kernel, the convolutional neural network is constructed

network to approximate partial differential equations, and use neural networks or other machine learning method to determine nonlinear response terms. When the partial differential equation corresponds to

When the following format

J f/ (f. .\'. \”) ' (.W, ÿ-. ff t. fJ \ . /J t \'. fJ \.\ . /J tg', - - ')

Do Euler discretization on the time derivative term, and reduce the space class derivative term

Beam convolution approximation, equation can be expressed as

$$\hat{u}(t_{i+1},\cdot) = D_0 u(t_i,\cdot) + \delta t \times (c_{00} D_{00} u +$$

$$c_{10} D_{10} u + c_{01} D_{01} u + c_{01} D_{01} u, \cdots)$$

inD and *D; j*

Represents the convolution kernel *C; j*

, is a two-dimensional matrix, equivalent tO

a partial

The coefficient of the derivative term can be obtained by interpolation, or directly in the training process

Learn from , and then judge the corresponding item according to whether the matrix is zero or not exists, so as to invert the unknown partial differential equation.

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Gonzàlez-Garcia et al. [2] based on artificial neural network system
The structure proposes a method for establishing a physical model, the essence of which is to
With empirical knowledge, list all the partial differential equations describing the physical process
Alternative, automatic selection and parameter estimation using artificial neural networks
In order to discover the hidden physical laws behind the data.
Wu et a1. [4] first constructed a residual network (ResNet) based on
A new framework for learning unknown differential equations from data.
based on the integral form inherent in the sub-equation to approximate the flow spectrum of the equation
(flow map, for ordinary differential equations) and evolution operators (evolution
operator, targeting partial differential equations), which fundamentally avoids
The numbers on which the traditional framework (which aims to approximate the right-hand side of the equation) relies on Value
differentiation.References [7] propose two multi-step ResNet
Through the network structure, from the perspective of the precise evolution operator, the first theoretical
The intrinsic mathematical relationship between ResNet and the exact evolution operator is established,
This gives a mathematical explanation of the deep learning method. Different from
Wu and Xiu [8] learn equations in modal/Fourier space, Chen et a1. [10] Learning and modeling in physical space, using DNN to learn to measure data
to learn unknown partial differential equations. Chen et al. [9] proposed a Gradient-Free Symbolic Genetic Algorithm (SGA-PDE), using symbolic math
Flexibly represent any given partial differential equation, optimize its representation, from
Open-form partial differential equations found in the data. Xu and Zhang

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[8]

A more robust deep learning based on PINN Genetic Algorithm (R-DLGA), which combines the deep learning-genetic algorithm provided by

Preliminary results of latent terms are added to the loss function as physical constraints to improve

The calculation accuracy of derivatives under the influence of high-order derivatives is improved, so that in

Obtain partial differential equations from noisy sparse data.

— N(u, x, t)

To date, many of the methods proposed in this field have some

Limitations. In particular, current methods usually study formal equations, but many physical equations are not in this category. Furthermore, if

How to eliminate evolutionary dynamics by measuring a system with parameter dependencies

The ambiguity between mechanics and its parameter dependencies is an open question Question. Although neural networks exhibit strong data learning capabilities,

for learning with noisy data, especially in nonlinear, multi-coupled complex.

In a complex physical system, the accuracy and stability of the network model need to be promote.

For a given given partial differential equation, the neural network can

The solution or characterization equation used to approximate partial differential equations, the next section of this paper will Fromthe three aspects of data-driven, physical constraints and physical-driven An introduction to solving partial differential equations by network, and a brief description of the methods used network, such as fully connected neural network, convolutional neural network (CNN),

Residual Network (ResNet), DenseNet,

Autoencoder Network

(autoencoder), long short-term memory (LSTM) networks, etc., summarize the present
There are important advances in research, and to explore the next development trend, for the future
To put forward suggestions on the research on intelligent solution of partial differential equations.

3 Deep learning-based partial differential equation solving method

3.1 Overview of Neural Network Solutions for Partial Differential Equations

x R

The basic structure of deep neural network is the feedforward fully connected deep neural network.
via the Internet [8]
, as an example to introduce the neural network of known partial differential equations
network solving method. Taking d- dimensional row vectors as the network input, a
The k-dimensional output form of a single hidden layer neural network is
8 in the formula in [w,b], represents the set of network parameters. The optimization of parameters adopts
Using Stochastic Gradient Descent (SGD) or its variants [9]
. by
Taking SGD as an example, the iteration process is as follows
a°*" = #" —›;v,›rf#*;x. ml
p and °8i
In the formula, is the step size of the th iteration. The loss function is relative to the model parameters
The gradient of numbers is usually computed using backpropagation [7], which is the reverse
A special case of the pattern automatic differentiation [48] technique. On Neural Networks
The optimization process will not be described in detail, please refer to [11] for details.

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I( )

B( )

For a given general partial differential equation, at the initial conditions

(IC) and boundary conditions (BC) can be expressed as [7].

$$\left.\begin{aligned} &\mathcal{N}(t,x;u(t,x;\theta))=0,\ \ t\in[0,\mathrm{T}],x\in\mathcal{D}\\ &\mathcal{I}(x;u(0,x;\theta))=0,\ \ x\in\mathcal{D}\\ &\mathcal{B}(t,x;u(t,x;\theta))=0,\ \ t\in[0,\mathrm{T}],x\in\partial\mathcal{D} \end{aligned}\right\}$$

in the formula u(t, x; 8) is the approximate solution of the equation, is the approximate solution in the equation

corresponding parameters N(-)

is a differential operator containing the time differential, empty

Linear or non-linear terms consisting of inter-differentials, etc.; x bounded connection continuous space domain D The position vector in , d D for the boundary.

Generally speaking, depending on the training method, based on deep learning

The partial differential equation solving methods can be divided into data-driven and physics-driven.

Two methods are used. The label data required by the data-driven method is in the form of u(t, x) By finding an optimal set of networkto parameters (w,b) to local

Minimize training data

U(t,X) and neural network prediction U+(t, x;W,b) difference. That is, the data-driven optimization problem can be formulated

shown as

In. .vqg (IV, It) = Jr(I,.i) — ii'(r, .r: IN, 6)JJ

lY', *b'* = art initi Jr,isg (lY, é)iv

where IV'

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and *b'ave* the optimization goals of the neural network.

For physically driven methods, governing equations and ICs are usually used,

BC constructs the residual, and then adds the residual to the loss function, thus Optimize the network parameters. The governing equation constructs the residual, and then optimizes the parameters by minimizing the residual.

The optimization problem based on physical constraints is as follows

$$loss_{psy}(\mathbf{W},\mathbf{b}) = \|\mathcal{N}(t,x,\hat{u}^*)\| + \|\mathcal{B}(t,x,\hat{u}^*)\| + \|\mathcal{I}(0,x,\hat{u}^*)\|$$

$$\mathbf{W}^*,\mathbf{b}^* = \arg\min_{w.b} loss_{psy}(\mathbf{W},\mathbf{b})$$

3.2 Data-driven neural network solution method for partial differential equations

When the partial differential equation is known, the partial differential equation is solved based on data-driven

Differential equations, whose core problem is to explore equations and their differentials The characterization method of the operator, and then the solution of the equation is obtained.

3.2.1 CNN-based solution method

Since the constrained convolution kernel mathematically has partial differential operator properties

It is very effective to use it to characterize and solve partial differential equations.

Therefore, the idea of characterizing differential operators based on restricted convolution kernels

want [7]

, Zha et al. [3] extended the 2D restricted convolution kernel to the 3D restricted convolution kernel

Convolution kernel to build a new intelligent solution method for three-dimensional partial differential equations.

For the convolution kernel q its moment matrix:

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$M(q) = (m_{i,j,t})_{N\times N\times N}$,

$$m_{i,j,t} = \frac{1}{i!\,j!\,t!} \sum_{k_1,k_2,k_3=-\frac{N-1}{2}}^{\frac{N-1}{2}} k_1^i k_2^j k_3^t q[k_1,k_2,k_3],$$

N — 1. When the maximum possible order of the three-dimensional differential operator is 1, in the convolution

nIn the process of kernel characterization differential operator, when the maximum order is constrained, q

For example I+j+t=1, for the restricted convolution kernel approximate differential operator we have

$$= \frac{1}{8}\begin{pmatrix} A_2, \\ A_2 \end{pmatrix}, q_{010} = \frac{1}{8}\begin{pmatrix} A_3, \\ A_3 \end{pmatrix}$$

x «

$$q_{001} = \frac{1}{}\Big(A_1,$$

1 1

$$\begin{matrix} 1 & -1 \\ 1 & -1 \end{matrix}$$

$$A_1 = \Big(\; 1 \quad 1 \qquad A_2 =$$

$$_3 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \qquad A_4 = \begin{matrix} -1 & -1 \\ -1 & -1 \end{matrix}$$

$$\frac{2}{\varepsilon}(q_{100}[-\cdot]\otimes u)[x,y,z] =$$
$$\frac{1}{4\varepsilon}(u(x,y,z) - u(x-\varepsilon,y,z)) +$$
$$\frac{1}{4\varepsilon}(u(x,y-\varepsilon,z) - u(x-\varepsilon,y-\varepsilon,z)) +$$
$$\frac{1}{4\varepsilon}(u(x,y,z-\varepsilon) - u(x-\varepsilon,y,z-\varepsilon)) +$$
$$\frac{1}{4\varepsilon}(u(x,y-\varepsilon,z-\varepsilon) - u(x-\varepsilon,y-\varepsilon,z-\varepsilon)) \approx$$
$$u_x(x,y,z), \qquad \text{as } \varepsilon \to 0$$

Similarly

$$\frac{2}{\varepsilon}(\boldsymbol{q}_{010}[-\cdot]\otimes\boldsymbol{u})\approx u_y(x,y,z),$$

$$\frac{2}{\varepsilon}(\boldsymbol{q}_{001}[-\cdot]\otimes\boldsymbol{u})\approx u_z(x,y,z).$$

Obviously, the convolution kernel under the constraint can be used to approximate the corresponding order

Differential operators, for partial differential equations

N(f,.v. v, z: ii) .= iq — F(a-. i', ?,times [7]

The added layered adaptive activation function can improve the training by 10 times

The training speed is improved, and the local error is improved. But at this time, 3D-PDE-Net

Not explicitly interpretable.

i r , . i r ,-. zz — . z,z , .

When performing convolution approximation, we have

(fj¿], “) = *D[j u“)*

$$\delta t\times F(x,y,z,\boldsymbol{D}_{000}\boldsymbol{u},\boldsymbol{D}_{100}\boldsymbol{u},\cdots)$$

On this basis, the hierarchical adaptive activation function is introduced to construct

3D-PDE-Net, as shown in Figure 2, where the activation function can be expressed as

‹r{ini'!D; j ii) , ii N I

is a predefined scale factor, the parameter is the excitation

The slope of the active function, the learnable parameter, uses the Tanh activation function.

Numerical experiments show that the solution accuracy of 3D-PDE-Net

L error

20 lower than solver ratio numerical format Douglas-Gunn ADI

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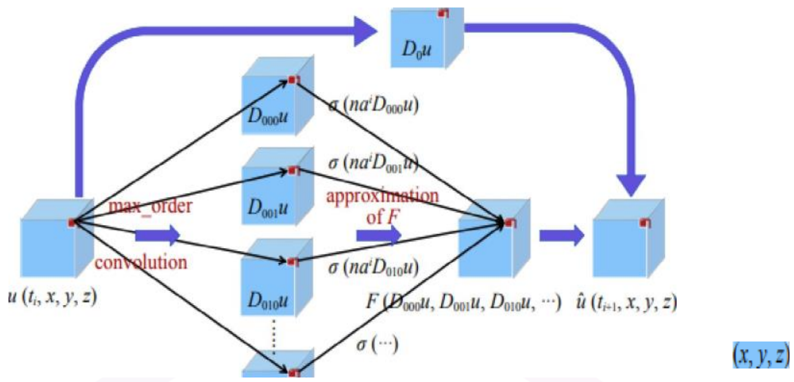

Figure 2 Schematic diagram of 3D-PDE-Net

3.2.2 Partial Differential Equation Solving Methods Based on Other Networks

Liu et al. [6] discussed the use of fully connected neural networks in function approximation.and proposes a general basic differential equation solver,

The initial value problem and boundary value problem of equations are mainly used for automatic differentiation.

Row solution. E et al. [3] and Han et al. [7] used deep learning to approximate the gradient

sub, based on the discrete scheme of partial differential equations, for high- dimensional partial differential equations

Give a new solution for deep learning. For H hidden layers, N

The network structure of the semi- linear parabolic partial differential equation with time interval is as follows

As shown in Figure 3, the entire network has a total of(HU)(N-1)

through loss function together to optimize all network parameters

$$t = t_1, t_2, \cdots, t_{N-1}$$

Middle Each column corresponds to a sub-network at a time step

$$h_n^1, h_n^2, \cdots, h_n^H$$

Interneurons in each sub-network.

Based on label data, use multiple

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The gradient operator is approximated by a layer feedforward neural network, which can be obtained higher than100-dimensional partial differential equation solution, and gives many types of high-dimensional partial differential

The result of solving the partial equation equation

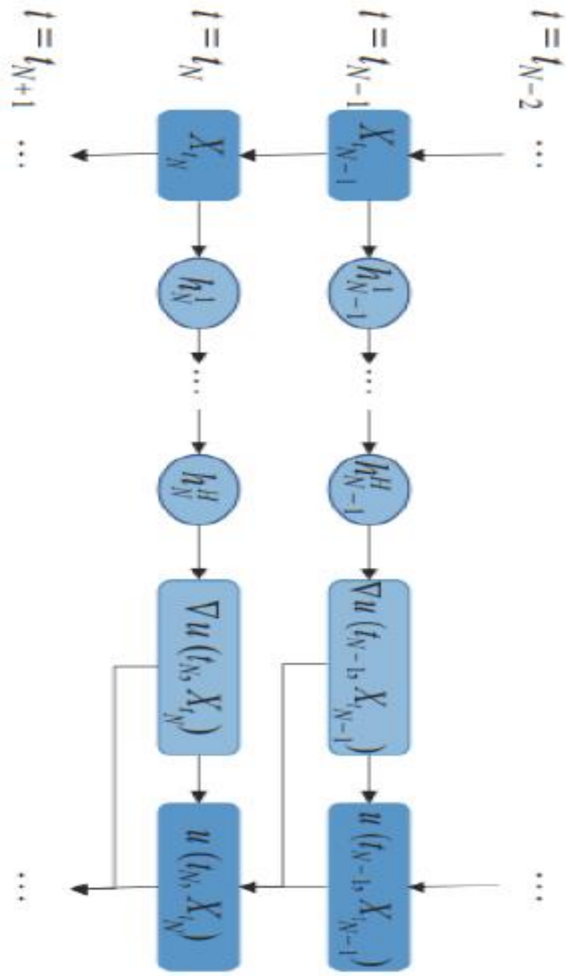

Figure 3. The semi-linear parabolic partial differential equation with H hidden layers and N time intervals network structure

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3.3 Partial Differential Equation Neural Network Solving Method under Physical Constraints

Due to the shortcomings of data drive such as weak generalization ability, physical drive

Dynamics can improve generalization and reduce labeled data. Physical drivers and data

Driven phase fusion, that is, the method of physical constraints, has received extensive attention.

In recent years' research[7]

, has seen the use of structured

Prior information builds machine learning algorithms based on data and physical information

[9] gave an analogy to LSTM human

deep Galerkin method (DGM) network of artificial neural network, providing

A calculation method of the second- order differential operator based on the Galerkin method is presented,

At the same time, the neural network approximation theorem under physical constraints is given.

3.1 PINN

Raissi et al. [2] used the governing equations of partial differential equations andIdentities such as boundary conditions are used to construct residuals, and the sum of residuals is used to construct residuals.

create a loss function, and extend this method to solve nonlinear problems. A neural network under physical constraints (physics informed neural network, PINN). PINN combines data-driven with physical constraints Therefore, a new idea of establishing and solving partial differential equations is proposed, that is,

pair partial differential equations

$$D(u(x)) = 0, \quad x \in \Omega$$
$$B(u(x)) = f(x), \quad x \in \partial\Omega$$

The loss function in PINN mainly consists of three parts, namely

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Governing equations of partial differential equations, network output and initial conditions, edges

Residuals for labeled data with bounded conditions.

At the same time, Raissi et al. [4] also studied the continuous time model and separation

Application of discrete time model in equation solving and equation recovery scenarios

and the influence of label data noise on solution accuracy and error propagation.

Taking the solution of Burgers equation under the Dirichlet boundary as an example, its

The equation is

$$u_t + uu_x - (0.01/\pi)u_{xx} = 0,\ x \in [-1,1],$$
$$u(0,x) = -\sin(\pi x)$$
$$u(t,-1) = u(t,1) = 0$$

there is a function D(u) for

$$D(u) := u_t + uu_x - (0.01/\pi)u_{xx}$$

The loss function can be defined as

$$L=\frac{1}{N_D}\sum_{n=1}^{N_D}\|D(\widehat{u}(x_n^D;W,b))\|^2+\frac{1}{N_u}\sum_{n=1}^{N_u}\|\widehat{u}(x_n)-u\|^2$$

The neural network parameters are optimized by minimizing the loss function such that

The network output approximates the solution to Burgers' equation.

3.3.2 Improved method based on PINN

Based on the PINN algorithm, Toshiyuki et al. [7] used three

The PINN framework composed of DNNs has an impact on the Richardson-Richards equation.

Parameter inversion was carried out using the program, and the water retention curve and hydraulic transfer function were estimated.Han et al. [1] introduced a deep learning-based general high- dimensional

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The solution method of the partial differential equation of matter.

Reconstruct, and then use the neural network to approximate the gradient of the unknown solution, in non-

Satisfactory numerical results were obtained in the calculation of linear equations. Meng et al. [8]

An improved PINN method, called PINN, is proposed by combining a

The long-term problem is decomposed into multiple independent short-term problems to add

The solution of the velocity partial differential equation. Michoski et a1. [1] studied the shock partial Differential Equation Neural Network Solving Method, Neural Network Method and Traditional

The comparison of the method results shows that,

The solution method based on neural network has excellent

potential, label data can effectively improve the solution accuracy. Kani and Elsheikh[10]

Solving PDEs with Physical Constraints and Orthogonal Decomposition (POD)

Combined with the discrete empirical interpolation method (DEIM), it provides a high

Accurate reduced-order models of nonlinear dynamical systems, reducing high-fidelity numbers

Computational complexity of the value simulation.

Jagtap et al. [8] proposed an adaptive activation function, which effectively improves the

Improve the efficiency of PINN approxin 10.0 7.5 ting nonlinear functions and partial differential equations,

Robustness and accuracy, the adaptive activation function is as follows, Figure 4 is for each

Fit the image of the activation function

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$$\left.\begin{aligned} &\text{sigmoid}: \frac{1}{1+e^{-ax}} \\ &\tanh: \frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}} \\ &\text{ReLU}: \max(0, ax) \\ &\text{leaky ReLU}: \max(0, ax) - v\max(0, -ax) \end{aligned}\right\}$$

However, the partial differential equation neural network solution with labeled data

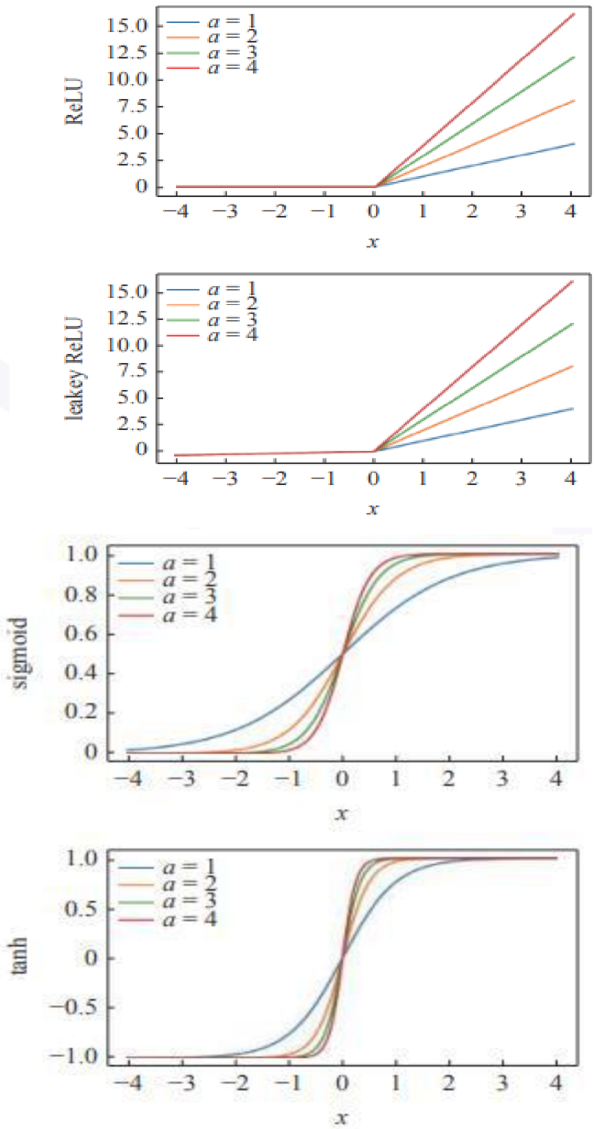

Figure 4. The corresponding variable a of Sigmoid, tanh, ReLU and leaky-ReLU activation

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method, there are great limitations. For many problems, the exact solution is

Unknown. If an exact solution of the partial differential equation is required to construct the loss

function, which greatly limits its scope of application. For example, in oil field development

During the process, the instrument can only measure the pressure at the bottom of the well and the production at the wellhead, while the

Can't get pressure from other regions. This means based on tag data

The partial differential equation solving method for is invalid. Therefore, based on purely physical constraints

(i.e., physics-driven) solution methods have broader application prospects,

Has the same convenience as traditional solvers (without any labels

data). Once this breakthrough, it will lead to the development of partial differential equation solving technology. real change

Partial differential equation neural network calculation under measurable label data
solution method

The above data-driven partial differential equation solving methods often require

For unknown quantities of distribution data, for example, you need to know the spatial distribution of pressure

data. This is often obtained under experimental conditions. For example, it can be

When multiple pressure sensors are arranged in the test, the pressure space can be obtained.

Variable data. But for practical engineering problems, this part of the data is not measurable

For example, in reservoir development, only the pressure in the well can be measured, other

The pressure data of are not available. Therefore, the above data- driven
It is difficult for the intel...

Wang et al. [8] combined the expert experience, physical

The integration of regular and sparse observation data into a

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theoretically guided neural network

(theory-guided neural network, TgNN), as shown in Figure 5, benefits Use TgNN to solve problems such as subsurface flow modeling,
uncertainty quantification, etc.

[5] used a deep neural network to solve the single-phase seepage problem,

Adding some measurable bottom hole flow pressure data as labels, effectively improving

Accurate solution of highly unstable partial differential equations with source and sink

degree. The biggest feature of this method is that dividing the observable bottom hole pressure number

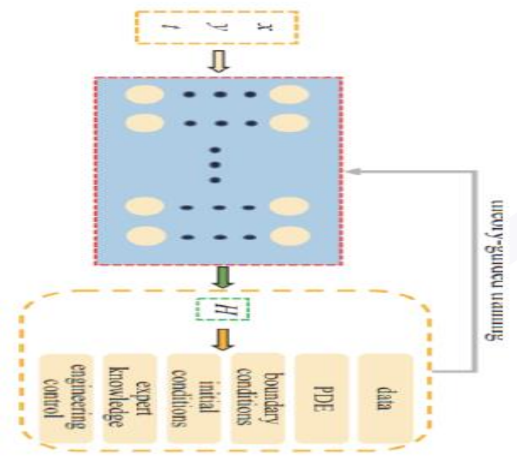

Figure S Network structure of the TgNN model

Chen et al. [8] proposed a covariance matrix-based optimization without

Gradient neural network, effectively improve the robustness of learning small data samples,

It is suitable for practical engineering applications. In the follow-up research, Chen et al. [9] proposed
A hard constraint projection (HCP)
of

The method improves the learning ability of machine learning methods for small sample data

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According to the label, there is no

need for any other label data, but The label data of the pressure distribution is replaced by the PDE constraints, thus

The practical application feasibilityof this method is greatly improved. In addition, using the source-sink term The induced gradient feature constructs a gradient model, which is added as a "signpost" to the After the network, by adding fixed neurons to help the network improve

optimization ability, and put forward the solution of pre-training to obtain "road signs"

Lu. Fig. 6 shows the pressure distribution, bottom hole pressure obtained by intelligent solution

(BHP) diagram.

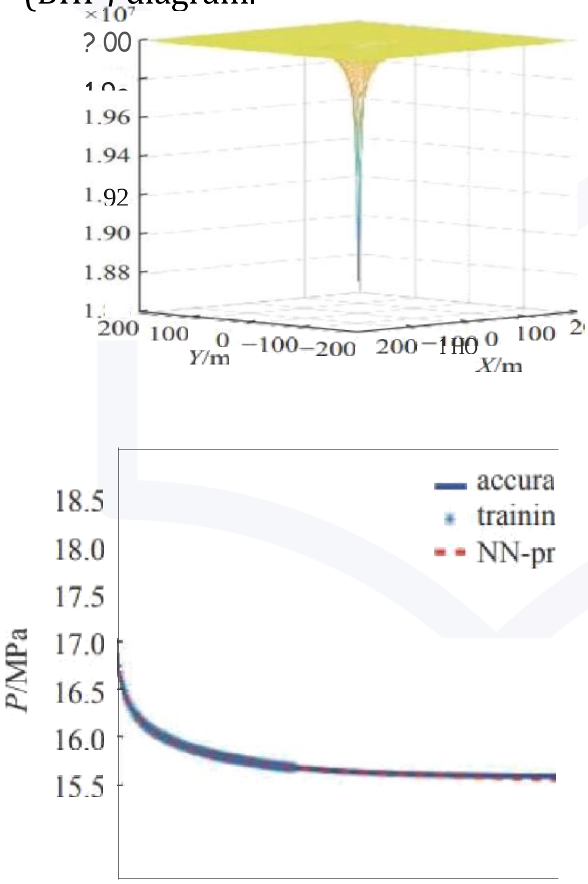

Fig. 6 Pressure distribution and bottom hole pressure nap obtained by intelligent solution

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Conclusion

From inverting unknown partial differential equations and solving known partial differential equations

From two perspectives, this paper summarizes the intelligent solution method of partial differential equations.

Development history, from data- driven, physics-constrained and physics-driven 3

In the aspect, the intelligent solution method of known partial differential equations is introduced,

The application scenarios and future research directions are briefly introduced. The mathematics community pays more attention

Research on the intelligent solution method of heavy general partial differential equations, combined with the

The applied research in the field of physics is being paid attention. If we can break through the physics

The bottleneck of the driving solution method is expected to subvert the traditional numerical partial differential equation

The solution technology has led to a huge change in numerical simulation technology.

Deep learning to solve partial differential equations has deep scientific

It is necessary to integrate deep learning theory, numerical simulation technology, partial differential

Mathematical essence of equations, physical meaning of partial differential equations and engineering background, etc.

Organic integration and deep crossover can achieve clear physical meaning and mathematical

A new method for solving partial differential equations with solid foundation and capable of solving engineering problems

law, which will promote mathematics, mechanics, artificial intelligence and reservoir engineering

Discipline integration and discipline development

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