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EFFECT OF NUCLEON TYPE ON ENERGY LEVELS AND REDUCED TRANSITION PROBABILITY OF ELECTRIC QUADRUPOLE FOR ^{30}Si and ^{30}S NUCLEI USING MODIFIED SURFACE DELTA INTERACTION

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Abstract
The nuclear structure of the nuclei ^{30}Si and ^{30}S was studied by nuclear shell model, using modified surface delta interaction MSDI in the model space (2S1\2 and 1d3\2) and the effect of the nucleon type on each of energy levels E_x and reduced transition probability of electric quadrupole B(E2) was demonstrated. Since each of two nuclei has two nucleons (two neutrons for ^{30}Si nucleus and two protons for ^{30}S nucleus) outside the closed core ^{28}Si. The values of each of angular momentum J, parity π, and excitation energy E_x were found for the permissible levels. As well as the reduced transition probability of electric quadrupole B(E2). From the calculations of the energy levels for each of angular momentum and parity, we obtained a perfect match for the level ground, and an acceptable conformity of the other levels with the experimental values.

Keywords: Energies levels, Reduced transition probability of electric quadrupole, 2S1\2 and 1d3\2 model space, Nuclear shell model, MSDI, Mirror nuclei.

1. Introduction
W. Elasser was the first to believe in the existence of a closed nuclear shell in 1934 [1]. Studies have shown that the nucleons inside nucleus is arranged in form of orbitals as atomic orbitals. It call levels structure or shell structure. The stability of nucleus is due to presence of some closed shells. The nuclear shell model assumed that nucleons move independently of each other, movement of each nucleon is independent on the movement of other nucleons [2]. It moves freely within the nucleus [3]. According to nuclear shell model, nucleons are distributed on separate energy levels, and its movement is within a nuclear potential that it

was able to form by itself, this potential is responsible for each nucleon movement separately [4,5].

The types of nuclei in shell model can be divided into three types [6] ;

1- Nuclei closed shell; this type of nuclei contains two shells, one of which is an inner shell that is completely filled, and the other outside is completely empty.

2- Single particle nuclei; shell of these nuclei is completely closed, except for shell, which lies in lowest energy level, it contains only one particle.

3- Single hole nuclei; shell of these nuclei is completely filled with the exception of one shell, it need only one nucleon.

The nuclear shell model is considered one of most important nuclear models that explained basic nuclear properties. It relied on the atomic shell model in many aspects. Where the experimental facts showed that the nuclei stability depends on atomic number (number of neutrons and protons in nucleus), so the nuclei are more stable when the number of protons or neutrons is equal to a magic number {2, 8, 20, 28, 50, 82, 126} [7].

The shell model depends on a preliminary model called (Single particle model). The single particle model calculations process depends on two basic principles [1];

1- Each nucleus moves freely in the force space, which is called nuclear potential, is the redial distance from nucleus center.

2- The shell (energy levels) is fill according to Pauli exclusion principle.

Where the single particle model was able to find spin and parity values for ground state and excited states [8]. it was able to achieve many successes that describe the nuclear features. Such as description of stable nuclei whose shell is closed and contain one or more nucleons [9]. it also explained the nuclear properties with valence nucleons, which are represented by energy levels, magnetic quadrupole moment, and electromagnetic transition probability resulting from radioactive decay [10].

Harmonic oscillator potential is one of easiest potentials that give approximate results for nuclear calculations. Especially for light nuclei and given by [11, 12];

$$V(r) = \frac{1}{2} M_p \omega^2 r^2 \tag{1}$$

where M_p is mass of a proton, ω is angular frequency, and r is distance between the coordinate center and each nucleon. It became clear from using of harmonic oscillator equation that there are orbits that are repeated in addition to the numbers of nucleons {2, 8, 20, 40, 70} may be a stable group. However, these numbers do not agree with experimental numbers, and for this reason, the interaction of spin and orbit was introduced [13]. The Saxon-Woods potential is considered according to equation below [14];

$$V(r) = \frac{-V_\circ}{1+\exp\left(\frac{r-R}{a}\right)} \tag{2}$$

where a is nuclear shell thickness and measured in ($a = 0.7$ fm), R is nucleus radius $R = 1.4\sqrt[3]{A}$ fm, A is mass number in units (amu).

2. Theory

Energy levels E_x and reduced transition probability of electric quadrupole B(E2) need to be calculated by applying nuclear shell model and using modified surface delta interaction. To determine the region in which these calculations are made, i.e. knowing model space in which the nucleons that represent the valence levels and appropriate interaction are distributed to them. The shell model calculations include:

1- Single particle energy which is symbolized by SPE.

2- The interaction between the two particles which is represented by two body matrix elements (TBME) [15], and that nucleons remain constant in the closed shell, they do not change, so the Hamiltonian will be responsible for dynamics of valence nucleons (protons - neutrons) by applying the shell model according to [11,16,17],

$$H = \sum_i n_i E_i + \sum_{ijkl} V_{ijkl}\, a_i^\dagger a_j^\dagger a_i a_j \tag{3}$$

where E_i is single particle energy of level i and its value can be found near the closed shell $A = A_{core} + 1$, n_i is number of nucleons (protons - neutrons) in level i, V_{ijkl} is matrix elements of the effective interaction between (nucleon – nucleon) for levels (i, j, k, l) can be found according to the relationship $A = A_{core} + 2$, $a_i^\dagger a_j^\dagger$ is creating or raising operators and $a_i a_j$ is annihilation or lowering operator. Modified surface delta interaction (MSDI) can be obtained from the derivation and modification of surface delta interaction (SDI) and by knowing the general properties of the SDI, the properties of MSDI were extracted [18, 19],

$$V^{MSDI}(\mathbf{r_1}, \mathbf{r_2}) = -4\pi A_T \delta(\mathbf{r_1} - \mathbf{r_2})\delta(\mathbf{r_1} - \boldsymbol{R}) + B(\tau_1 \cdot \tau_2) + C \tag{4}$$

where $\mathbf{r_1}, \mathbf{r_2}, \tau_1,$ and $\tau_2$location vectors, A_T, B, and C is MSDI coefficients, **R** is the radius of the nucleus, and T is isospin.

$$\langle \tau_1 \cdot \tau_2 \rangle_T = 2T(T+1) - 3 \tag{5}$$

$$\langle B(\tau_1 \cdot \tau_2) + C \rangle = \begin{cases} -3B + C & \text{for } T = 0 \\ B + C & \text{for } T = 1 \end{cases} \tag{6}$$

$$A_T \approx B \approx \frac{25}{A} \text{ and } C \leq 0 \tag{7}$$

Hamiltonian matrix elements of two nucleons can be found [20, 21],

$$H_{ij} = (\varepsilon_i + \varepsilon_j)\delta_{ij} + \langle j_a j_b | V^{MSDI}(\mathbf{r_1}, \mathbf{r_2}) | j_c j_d \rangle_{J,T} \tag{8}$$

$$\langle j_a j_b | V^{MSDI}(\mathbf{r_1}, \mathbf{r_2}) | j_a j_b \rangle_{J,T} = -A_T \frac{(2j_a+1)(2j_b+1)}{2(2J+1)(1+\delta_{ab})} \left\{ \left\langle j_b - \frac{1}{2}\, j_a \frac{1}{2} \middle| J0 \right\rangle^2 \left[1 - (-1)^{\ell_a + \ell_b + J + T}\right] + \left\langle j_b \frac{1}{2}\, j_a \frac{1}{2} \middle| J1 \right\rangle^2 \left[1 + (-1)^T\right] \right\} + [2T(T+1) - 3]B + C \tag{9}$$

If $T = 1$, then;

$$\langle j_a j_b | V^{MSDI}(\mathbf{r_1}, \mathbf{r_2}) | j_a j_b \rangle_{J,1} = -A_1 \frac{(2j_a+1)(2j_b+1)}{2(2J+1)(1+\delta_{ab})} \left\langle j_b - \frac{1}{2}\, j_a \frac{1}{2} \middle| J0 \right\rangle^2 \left[1 + (-1)^{\ell_a + \ell_b + J}\right] + B + C \tag{10}$$

where $\langle j_a j_b | V^{MSDI}(\mathbf{r_1}, \mathbf{r_2}) | j_a j_b \rangle_{J,T}$ represents matrix elements, j_a and j_b are orbits, J is total angular momentum, T total isospin, and $\left\langle j_b - \frac{1}{2}\, j_a \frac{1}{2} \middle| J0 \right\rangle$, $\left\langle j_b \frac{1}{2}\, j_a \frac{1}{2} \middle| J1 \right\rangle$ are Clebsch-Gordon coefficients. The nuclei remain in excited state in nuclear reactions and radioactive decay, the decomposition of the excited state into the ground state is done by emitting photons (gamma rays) whose energy is difference between two states [22]. The energy levels were determined by knowing energy states and gamma ray spectrum, which led to emergence of nuclear models. gamma ray is electromagnetic radiation emitted from the excited nucleus. It can be assumed that the nucleus consists of point nucleons. It has a dipole magnetic moment. It has a charge as proton, and the charges may be distributed. it can interact with the external field so it will cause electric transitions (EL). As for magnetism, it is subjective for each nucleon. In addition to the magnetism arising from the movement of nucleons (protons) in closed orbits. So magnetic-transitions (ML) can occur in which case the spin will change. This is because gamma rays carry an angular moment estimated at $\hbar J$ [23]. The probability of decay of the nuclear states can be calculated through the direct information of two states (initial and final), and thus distorted structures will be

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revealed inside the nucleus [22]. When the nucleus excited, it can decay by emitting alpha or beta particles. It splits into another nucleus. Or descends to ground level. It emits gamma rays. The reason for this transition is due to the interaction of the excited nucleus and the external electromagnetic field [24]. The is reduced transition probability of electric multipole B(EL) is given by [25];

$$B\left(EL:\ J_i \longrightarrow J_f\right) = \frac{2J_f+1}{2J_i+1}\left|\langle J_f||\hat{Q}_{EL}||\,J_i\rangle\right|^2 \qquad (11)$$

where $\hat{Q}_{EL}$ is electric multipole operator, J_i and J_f are initial and final total angular momentum.

3. Results and Discussion

Energy levels and reduced transition probability of electric quadrupole were calculated theoretically for ^{30}Si and ^{30}S nuclei, using modified surface delta interaction (MSDI) for arrangement of pure and mixed configurations in (2S1\2 and 1d3\2) model space where each of them contains two particles outside the closed core ^{28}Si. There are three possibilities of two nucleons, then the values of total angular and parity will be as follows:

Table 1: The cases of nucleons distribution in the model space.

Cases	J^{π}
$(2S1/2)^2$	0^+
$(2S1/2)^1(1d3/2)^1$	$1^+, 2^+$
$(1d3/2)^2$	$0^+, 2^+$

For calculate the energy levels associated with above cases for both pure and mixed configuration of two neutrons in ^{30}Si nucleus and two protons in ^{30}S nucleus. Equivalent in the space of form 2S1\2 and 1d3\2.

3.1 Energy levels

Using Eq. 8 the matrix elements were calculated for two neutrons and the modified surface delta interaction calculated by Eq. 10 with coefficients values A = 0.79 MeV, B = 0.654 MeV, and C= -2.0MeV for ^{30}Si core and A = 0.815 MeV, B = 0.75 MeV, and C= -0.3 MeV for ^{30}S nucleus. The theoretical calculations are Compared with the experimental data values of energy levels [26] for both nuclei as in Table 2 and Table 3.

Table 2: Comparison of energy levels theoretical results of ^{30}Si with experimental values.

Theoretical results				Experimental values [26]	
Pure configuration		Mixed configuration			
J^{π}	$E_x(MeV)$	J^{π}	$E_x(MeV)$	J^{π}	$E_x(MeV)$
0^+_1	0.0	0^+_1	0.0	0^+_1	0.0
1^+_1	2.06	1^+_1	2.06	1^+_1	3.74
2^+_1	1.43	2^+_1	1.85	2^+_1	2.2
0^+_2	1.76	0^+_2	2.84	0^+_2	3.78
2^+_2	3.02	2^+_2	3.68	2^+_2	3.49

Table 3: Comparison of energy levels theoretical results of ^{30}S with experimental values.

Theoretical results				Experimental values [26]	
Pure configuration		Mixed configuration			
J^{π}	$E_x(MeV)$	J^{π}	$E_x(MeV)$	J^{π}	$E_x(MeV)$
0^+_1	0.0	0^+_1	0.0	0^+_1	0.0
1^+_1	2.2	1^+_1	2.2	1^+_1	3.74
2^+_1	1.54	2^+_1	1.935	2^+_1	2.522
0^+_2	1.95	0^+_2	2.983	0^+_2	3.78
2^+_2	3.25	2^+_2	3.85	2^+_2	3.49

From two tables above for energy levels of two nuclei (^{30}Si and ^{30}S), For the pure configuration, we predict a perfect match between the theoretical value of ground level ($J^{\pi} = 0^+$) and practical value ($J^{\pi} = 0^+$) and an acceptable match for excited levels, while for the mixed configuration, we obtained better results for the excited levels, and the results improved in the mixed configuration. Energy levels calculations in mixed configuration are better than energy calculations in pure configuration and more compatibility with experimental values., as this is due to energy difference between two appearances that is in mixed configuration is greater than pure configuration.

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3.2 The reduced transition probability of electric quadrupole

Calculations showed effect of nucleon type in ^{30}Si and ^{30}S mirror nuclei on reduced transition probability of electric quadrupole B(E2) for transitions allowed within model space (2S1/2 and 1d3/2), where the eigenvectors values of matrix elements were calculated by Eq. 8. Then, the reduced transition probability of electric quadrupole B(E2) were calculated by Eq. 11, as shown in Table 4 and Table 5, for two nuclei respectively, which show the results of theoretical calculations for B(E2) and compare them with available experimental values [26].

Table 4: Comparison of reduced transition probability of electric quadrupole theoretical calculations for ^{30}Si nucleus with available experimental values.

$J_i^\pi \longrightarrow J_f^\pi$	**Theoretical results e^2fm^4**	**Experimental values e^2fm^4 [26]**
$2^+_1 \longrightarrow 0^+_1$	46.029832	47.06535
$2^+_2 \longrightarrow 2^+_1$	49.22	49.8339
$2^+_2 \longrightarrow 0^+_1$	7.46286	9.41307
$1^+_1 \longrightarrow 2^+_1$	8.4142	8.30565
$0^+_2 \longrightarrow 2^+_1$	8.05338	7.75194

Table 5: Reduced transition probability of electric quadrupole theoretical calculations for ^{30}S nucleus.

$J_i^\pi \longrightarrow J_f^\pi$	**Theoretical results e^2fm^4**
$2^+_1 \longrightarrow 0^+_1$	0.97756
$2^+_2 \longrightarrow 2^+_1$	81.94437
$2^+_2 \longrightarrow 0^+_1$	8.0957
$1^+_1 \longrightarrow 2^+_1$	5.72458
$0^+_2 \longrightarrow 2^+_1$	16.12682

From comparison results of theoretical calculations for B(E2) with available experimental values as in Tables 4 and 5 when conducting this study, we obtained a good agreement for the transition value B(E2: $2^+_2 \longrightarrow 2^+_1$) for the nucleus of ^{30}Si, while the experimental values aren't available for the nucleus of ^{30}S.

4. Conclusions

After knowing the characteristics of the nuclear shell model of ^{30}Si and ^{30}S nuclei and by calculations of excitation energy levels and reduced transition probability of electric quadrupole using modified surface delta interaction, it became clear to us the following:

1- There is an effect of nucleon type for each nucleus on the results of the excitation energy levels and reduced transition probability of electric quadrupole. There are two neutrons outside the closed core of ^{28}Si nucleus of ^{30}Si nucleus and two protons of ^{30}S nucleus.

2- We obtained a perfect match for energy levels between theoretical calculations and experimental data values, and an acceptable match for the rest of levels due to the few differences in A and B coefficients values that were chosen.

3- We found that the nuclear shell model achieved great success when using modified surface delta interaction (MSDI) to calculate the energy levels and reduced transition probability of electric quadrupole within model space 2S1/2 and 1d3/2 in this study.

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