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VAN HIELE THEORY AND ITS IMPORTANCE IN TEACHING GEOMETRY

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Abstract

This article focuses on the Van Hiele theory and its importance in geometry education. The theory explains the step-by-step development of students' geometric thinking and provides a methodological basis for selecting content, teaching methods, and learning sequences. The paper discusses the Van Hiele levels and instructional phases, assessment/diagnostic approaches, and a set of supporting problems and tasks aligned with each level.

Keywords: Geometric thinking, Van Hiele levels, instructional phases, visualization, proof, diagnostics, task system.

Introduction

Geometry is one of the fundamental components of mathematics education, contributing to the development of students' spatial imagination, logical reasoning, and proof skills. Nevertheless, many studies indicate that students often experience substantial difficulties in learning geometry. A major cause of these difficulties is the mismatch between the complexity of geometric concepts and students' age-related cognitive development and modes of thinking. Addressing such challenges requires a scientifically grounded instructional framework, and the Van Hiele theory provides a particularly important approach in this regard.

The Van Hiele theory, developed by Dutch mathematics educators Pierre van Hiele and Dina van Hiele-Geldof, describes the gradual development of geometric thinking through successive levels. The theory substantiates the need to select the content, teaching methods, and instructional sequence in geometry in accordance with students' cognitive readiness.

In practical experience within general secondary education, the following learning problems are frequently observed:

- A student recognizes a shape but cannot distinguish or articulate its properties.
- The student knows certain properties but does not understand the logical relations among them (e.g., conditional "if... then..." reasoning).
- The student memorizes theorems but cannot complete proofs independently and coherently.
- When the representation of a figure changes (e.g., through transformations or similarity), the student's understanding of the figure becomes unstable.

These difficulties are related not only to the inherent complexity of geometry but also directly to the learner's level of geometric thinking. The Van Hiele theory explains this issue from a scientific and methodological perspective: success in geometry depends largely on whether instruction is organized in a way that matches the learner's current stage of geometric reasoning.

Accordingly, this article presents a systematic account of the Van Hiele theory, demonstrates its practical value for lesson planning, and offers methodological recommendations enriched with level-based supporting problems.

Objectives of the study: 1) To explain the essence of the Van Hiele theory; 2) To characterize students' learning activity across levels (0–4 / 1–5); 3) To integrate the Van Hiele instructional phases into the lesson process; 4) To propose diagnostic and assessment approaches; 5) To present a system of tasks and problems corresponding to each level.

Literature Review

The Van Hiele model is a theory describing how students learn geometry. It emerged in 1957 in doctoral dissertations at Utrecht University by Dina van Hiele-Geldof and Pierre van Hiele. Researchers in the former Soviet Union conducted investigations on the theory in the 1960s and incorporated their methodological findings into curricula. American researchers in the late 1970s and early 1980s carried out several major studies of the Van Hiele theory, showing that low Van Hiele levels of geometric readiness make proof-related problem solving difficult, and recommending strengthening preparatory work from lower grades in this direction. The model significantly influenced geometry curricula worldwide, emphasizing both the analysis of properties and an increased focus on classification of shapes in early grades. In the United States, the theory also influenced geometry-related standards published by the National Council of Teachers of Mathematics and the Common Core.

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Research Methodology

This section discusses the essence and didactic idea of the Van Hiele theory. Its central premise is that geometric concepts and methods of proof are formed in the learner's mind through successive levels. There are no "jumps" between levels: learners can meaningfully adopt the language, proof practices, and abstractions of a higher level only after accumulating experience at the lower levels.

This theory shows "complexity" in geometry in two ways:

- Content complexity (topics, theorems, proof systems).
- Cognitive complexity (how learners interpret and reason about content using the language and tools available at their current level).

Therefore, rather than concluding that "the topic is difficult," the teacher should consider: "At which level can the learner understand this topic?"

Core principles of the Van Hiele levels:

- Sequential progression: levels are passed in order.
- Language principle: each level has its own "language of understanding" (from visual descriptions to property-based language to formal deductive language).
- Separation principle: students at different levels may interpret the same term differently.
- Instructional influence: advancement depends more on the quality and type of instruction than on age alone.

The Van Hiele levels (stages of geometric thinking) are given in practice in 5 forms. In some sources they are numbered 0–4, in others 1–5. Below the content is the same, but the numbers are given in the form "0–4".

➢ Level 0: Visualization (Recognition by Appearance)

At this level, students recognize geometric shapes based on their appearance.

Signs: The shape is recognized by its "general appearance." The student says "this is a square," "this is a triangle," but cannot justify why with properties.

Typical errors: perceiving a rhombus as a square or vice versa; imagining a parallelogram only as a "slanted rectangle."

➢ Level 1: Analysis (Identifying Properties)

At this stage, students begin to distinguish between the individual properties of geometric shapes.

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Characteristics: the student can enumerate the properties of shapes: "the angles of a right rectangle are 90°", "opposite sides of a parallelogram are parallel".
Limit: the logical "if-then" connection between the properties is still weak.

➢ **Level 2: Abstraction / Informal Deduction**

Students at this level begin to understand the connections between properties, and understand the logical relationships between definitions and theorems. Simple proofs and reasoning are performed, but a strict axiomatic system is not yet fully mastered.
Characteristics: the student sees the connection between properties: "if all angles in a rectangle are 90°, then it is a right rectangle". Understands the classification: "a square is both a right rectangle and a rhombus".

➢ **Level 3: Deduction (Formal Deductive Reasoning)**

At this level, students can work in a fully deductive system. They understand the strict logical connections between axioms, definitions, theorems, and proofs. This stage is important in the upper grades of school and at the academic lyceum level.
Characteristics: the student works in a chain of definitions, axioms, theorems; knows the structure of the proof (given–proved–evidence–conclusion).

➢ **Level 4: Rigor (Comparing Systems; Advanced Formalism)**

The highest level, at which the student can compare different axiomatic systems, understand different models of geometry. This level is formed mainly at the stage of higher education.
Characteristics: the student approaches the level of comparing different geometries (for example, Euclidean and non-Euclidean ideas), analyzing the significance of axioms.

Analysis and Results

The Van Hiele approach not only identifies levels but also proposes five instructional phases that guide learners from one level to the next. These phases are convenient for geometry lesson planning:

1. Information: introducing the topic, posing a problem, activating prior experience.

2. Guided orientation: teacher-led work with models, drawings, experiments.
3. Explanation: students express conclusions in their own words; terminology is formed.
4. Free orientation: independent application in new situations; more complex tasks.
5. Integration: final generalization and systematization; reflection (“What have we learned?”).

Methodological value of the Van Hiele theory in geometry instruction:

1) Selecting content and sequencing appropriately

According to the Van Hiele approach, instruction should proceed as: visual and hands-on activities → property analysis → relationships and informal proof → formal proof. If this sequence is violated, students tend to memorize proofs rather than understand them.

2) Visualization tools and modelling

Effective tools include cutting and pasting shapes, folding (origami), constructions; dynamic geometry software (e.g., “dragging” to test invariants); and illustrative drawings and 3D models.

3) Language development and mathematical communication

Learners’ language differs across levels. Teachers should not reject students’ informal language; instead, they should gradually guide it toward precise scientific terminology.

4) Differentiated instruction

Even within one class, students may be at different levels. Therefore, providing 2–3 types of tasks (basic–intermediate–advanced) is effective. Group work with distributed roles (drawer, explainer, verifier, summarizer) can also increase learning efficiency.

When assessing (level determination and monitoring progress), i.e. determining which of the above levels have been mastered and in what condition, the following types of questions allow you to quickly determine the level:

- Visual level: “Which pictures are rectangles? Why?” (usual and unusual cases).
- Analytical level: “Write three properties of a rectangle.”
- Abstraction (informal deductive): “If the diagonals of a rectangle are equal, is it necessarily a right rectangle? Give a counterexample.”

➢ Deduktiv: "Berilgan teoremani aksiomalar yoki oldingi teoremalar asosida isbotlang."

The following system of questions can be presented, based on the five levels mentioned:

➢ **Tasks for Level 0 (Visualization)**

Problem 1 (Recognition and grouping): Divide the given shapes into three groups: triangles, quadrilaterals, and circle-like figures. Name each group.
Focus: overall appearance, number of sides, "roundness."
Problem 2 (Cut-and-paste / folding): Cut any triangle from paper. How can it be constructed or folded so that an axis of symmetry appears?
Focus: visual approach to the concept of an isosceles triangle.
Problem 3 (Construction): Using given points, construct: (a) a triangle, (b) a quadrilateral, (c) a pentagon.
Focus: structure of shapes, construction skills.
Problem 4 (Real-life examples): Find five examples of geometric shapes in your surroundings (window, book, clock, sign, etc.).
Focus: connecting geometry with everyday life.

➢ **Tasks for Level 1 (Analysis)**

Problem 1 (Identifying properties): Write at least three properties of a rectangle and three properties of a rhombus. Which properties are common?
Focus: sides, angles, diagonals.
Problem 2 (True/false with justification): "Every square is a rhombus." Is this statement true? Justify using properties.
Focus: all sides of a square are equal.
Problem 3 (Diagonals of a parallelogram): State a property of the diagonals of a parallelogram and verify it using a diagram.
Focus: diagonals bisect each other.
Problem 4 (Measurement and observation): Draw different triangles and measure the sum of interior angles in each. What conclusion do you reach?
Focus: experimental approach to the 180° result.
Problem 5 (Completing a definition): Complete the definition: "A parallelogram is a …" (minimal and sufficient).
Focus: "a quadrilateral with opposite sides parallel."

➢ **Tasks for Level 2 (Abstraction / Informal Deduction)**

Problem 1 (Classification chain): Place a square into the classes of "rectangle" and "rhombus," explaining via properties.

Focus: satisfying both definitions.

Problem 2 (Counterexample reasoning): "Any parallelogram with equal diagonals is a rectangle." Is the statement correct? If correct, justify; if not, provide a counterexample.

Focus: informal justification (angle analysis).

Problem 3 (From properties to identification): If a quadrilateral's diagonals are perpendicular and bisect each other, what quadrilateral is it? Justify.

Focus: rhombus (perpendicular diagonals + parallelogram property).

Problem 4 (Medians of a triangle): Observe (via drawing/coordinates) that the medians of a triangle intersect at one point. In what ratio does this point divide each median?

Focus: approaching the 2:1 ratio idea via informal argument.

Problem 5 (Planning a proof): For the theorem "Base angles of an isosceles triangle are equal," what auxiliary construction would you choose?

Focus: using a median/segment to apply triangle congruence criteria.

➢ **Tasks for Level 3 (Deduction)**

Problem 1 (Formal proof): Prove that opposite angles of a parallelogram are equal.

Focus: angle relationships formed by parallel lines (alternate/corresponding angles).

Problem 2 (Theorem chain): Prove that if a quadrilateral is a parallelogram and its diagonals are equal, then it is a rectangle.

Focus: deriving right angles using triangle congruence.

Problem 3 (Locus): Find and prove the set of points in the plane equidistant from two given points A and B.

Focus: perpendicular bisector of segment AB.

Problem 4 (Circle theorem): Prove that the center of a circle lies on the perpendicular bisector of any chord (or prove the converse).

Focus: equal radii, triangle congruence.

Problem 5 (Analyzing a proof): In a given proof, identify which steps use: (1) an axiom, (2) a definition, (3) a previous theorem, (4) a conclusion.

Focus: proof culture and referencing sources.

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➢ **Tasks for Level 4 (Rigor) (optional/advanced)**

Problem 1 (Changing an axiom): If the parallel postulate changes, what can be hypothesized about the sum of the angles in a triangle? (Idea-level discussion.)

Focus: understanding axiom–consequence relationships.

Problem 2 (Comparing definitions): A rhombus can be defined as: (a) a quadrilateral with all sides equal; (b) a parallelogram with perpendicular diagonals; (c) a parallelogram with one pair of adjacent sides equal. Which are equivalent? Which conditions are sufficient/necessary?

Focus: equivalence, necessary vs. sufficient conditions.

Conclusion/Recommendations

In conclusion, the Van Hiele theory has substantial theoretical and practical value for geometry education. It explains how students' geometric thinking develops and provides teachers with a framework for organizing instruction effectively. Geometry lessons designed with explicit consideration of Van Hiele levels promote robust understanding, logical reasoning, and a culture of proof. The theory demands that instruction be aligned with learners' current levels of geometric reasoning. Consistent progression between levels, systematic incorporation of the instructional phases, increased emphasis on visual and hands-on activities, and the use of differentiated tasks all contribute to deeper and more meaningful learning of geometry. Hence, the Van Hiele theory serves as a key methodological foundation for planning geometry lessons because it:

➢ enables instruction to be organized with respect to students' levels of geometric thinking;

➢ supports the logical and sequential structuring of topics;

➢ encourages effective use of visual tools, models, and practical activity;

➢ develops logical thinking and proof skills gradually and systematically.

In particular, applying a Van Hiele-based approach in general secondary schools can increase students' interest in geometry and improve learning outcomes.

Finally, the system of level-based supporting tasks presented in this paper offers practical assistance to teachers for designing meaningful lessons, developing students' thinking step by step, and forming proof competence.

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